

***Optical Properties of Nonspherical  
Particles: Physical Models and Machine  
Learning***

**Lei Bi**

**School of Earth Sciences, Zhejiang University  
Hangzhou, China**

**Acknowledgment: Funding support from CMA, NSF. Contributions from postdocs and graduate students: Wushao Lin, Zheng Wang, Meng Li, Hejun Xie, Lanhui Sun, Jinhe Yu, Senyi Kong.**



# Zhejiang University (1897-)

“Seeking Truth, Pursuing Innovation”.

**Atmospheric Sciences (1936-)**

Contact: [bilei@zju.edu.cn](mailto:bilei@zju.edu.cn)





# **INTERNATIONAL RADIATION SYMPOSIUM**

**JUNE 2024, HANGZHOU, CHINA**

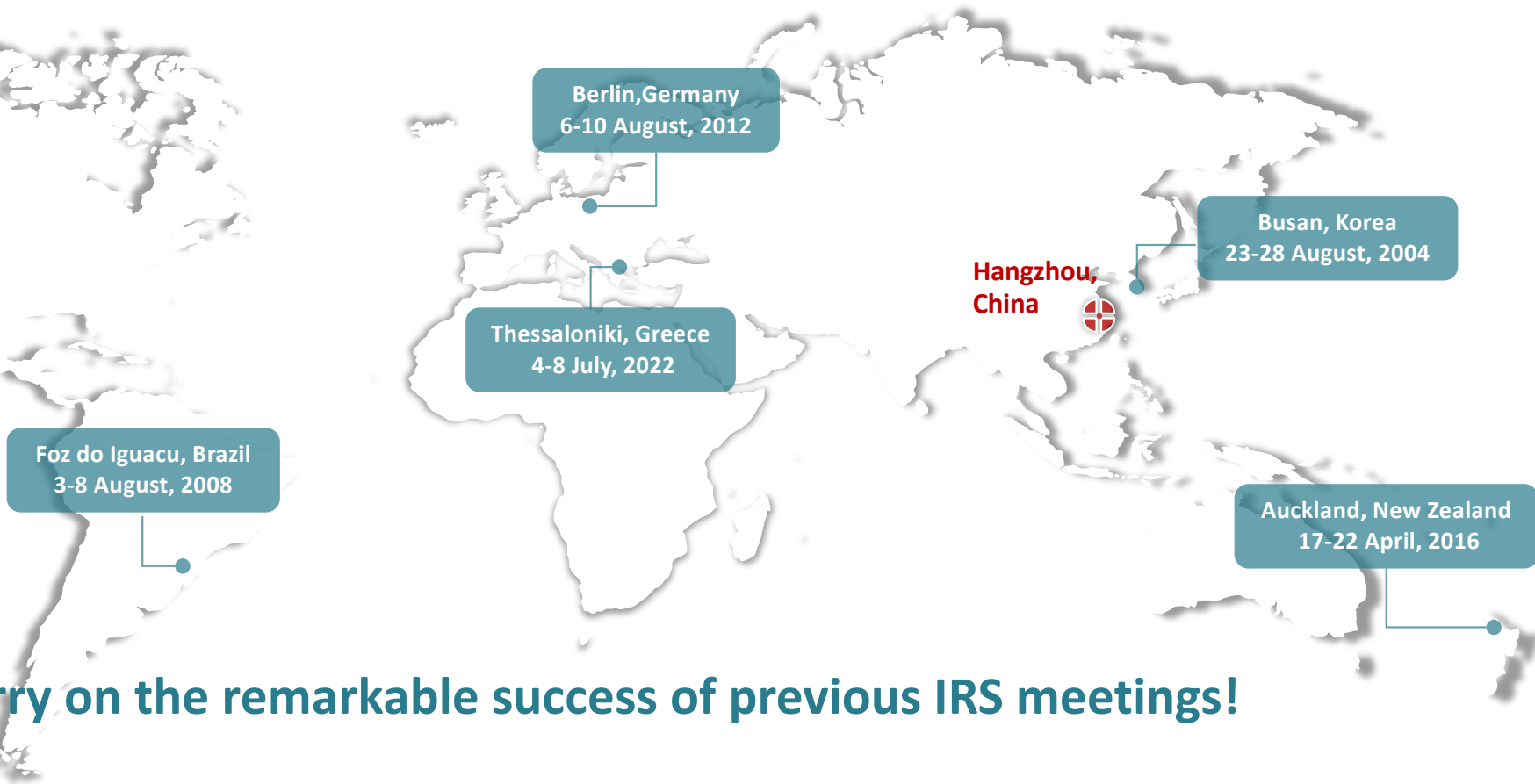
**Zhejiang University**

**Lei Bi (local host, IRC commissioner)**



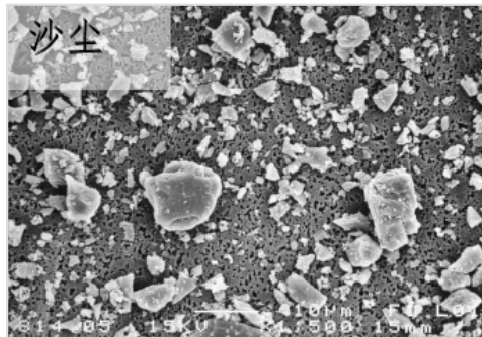


# Celebrate the 20-year Anniversary in Asia!

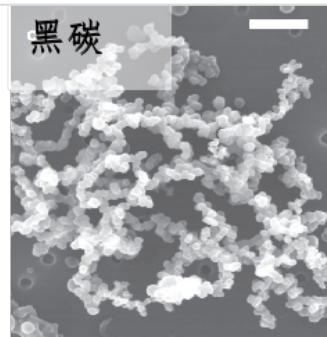


Carry on the remarkable success of previous IRS meetings!

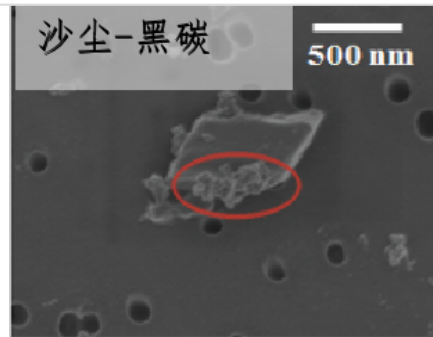
# Cloud, Aerosol and Precipitation Particles



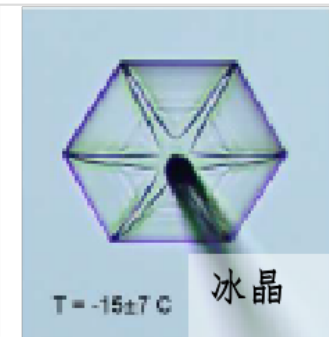
Volten et al., 2001



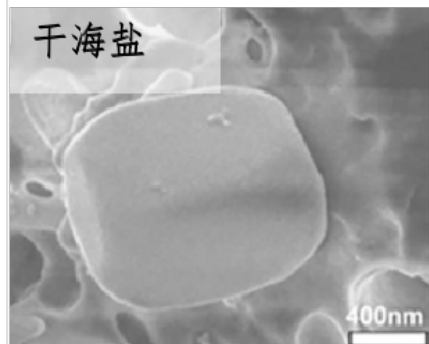
China et al., 2013



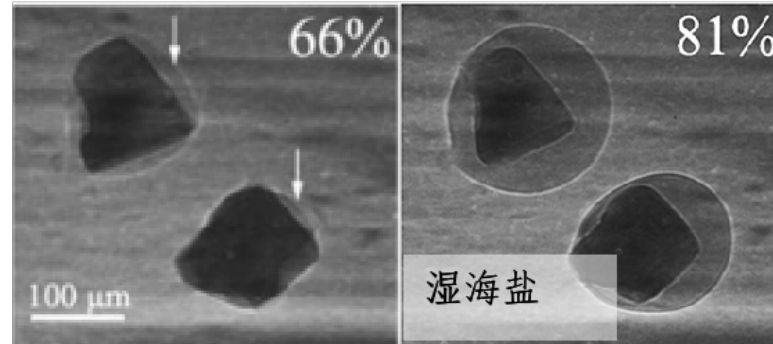
Scarnato et al., 2015



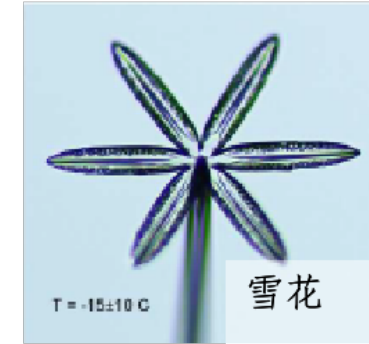
Libbrecht et al., 2001



Gwaze et al., 2007



Zeng et al., 2013



Libbrecht et al., 2001

**We hope that all particles are spheres, but they are not.**

# Van de Hulst: A long way ...

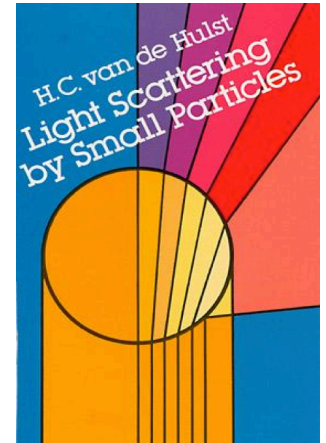
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## PREFACE

The scattering of electromagnetic waves by a homogeneous sphere is a problem with a known solution. I first met this problem when I needed some numbers and curves in an astrophysical investigation. I soon learned that it is a long way from the formulae containing the solution to reliable numbers and curves. Subsequent conversations and correspondence with other research workers, notably in chemistry, showed that the same difficulty was felt in other fields.

## Light Scattering by Small Particles (A “bible” of light scattering)



The story is also very true for nonspherical particles.

# Problems and Challenges

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- **Efficient and accurate computational methods**  
**arbitrary shapes, inhomogeneities and sizes**
- **Flexible particle representations (aerosols, hydrometeors)**  
**shapes, refractive indices , size distributions**
- **Datasets, parameterization, validation ...**  
**tremendous efforts**

# Progresses

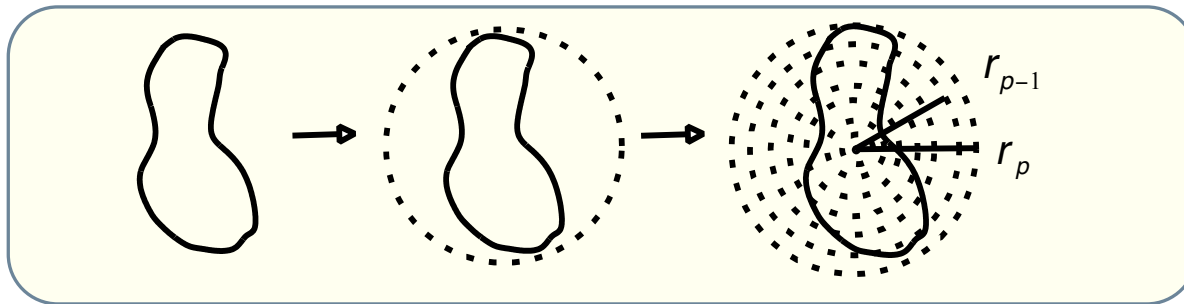
8

- **T-matrix, Super-formula, Machine Learning**



# Invariant Imbedding T-matrix

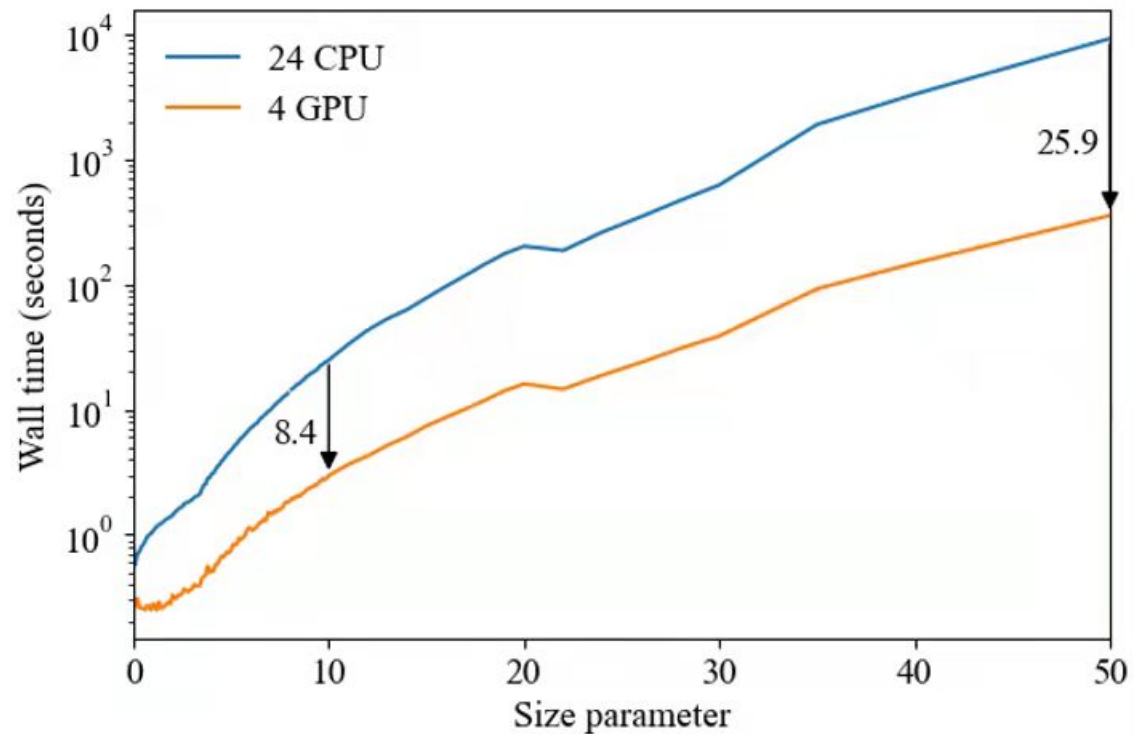
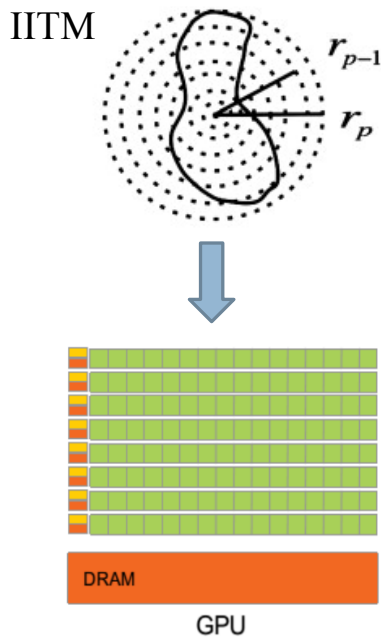
9



$$T_{mnmn'}(r + dr) = Q_{11}^m(r + dr) + \left[ \mathbf{I} + Q_{12}^m(r + dr) \right] \left[ \mathbf{I} - T_{mnmn'}(r) Q_{22}^m(r + dr) \right]^{-1} T_{mnmn'}(r) \left[ \mathbf{I} + \tilde{Q}_{12}^m(r + dr) \right]$$

(Johnson, 1988; Bi et al., 2013; Bi and Yang, 2014; Bi et al., 2018; Bi et al., 2022)

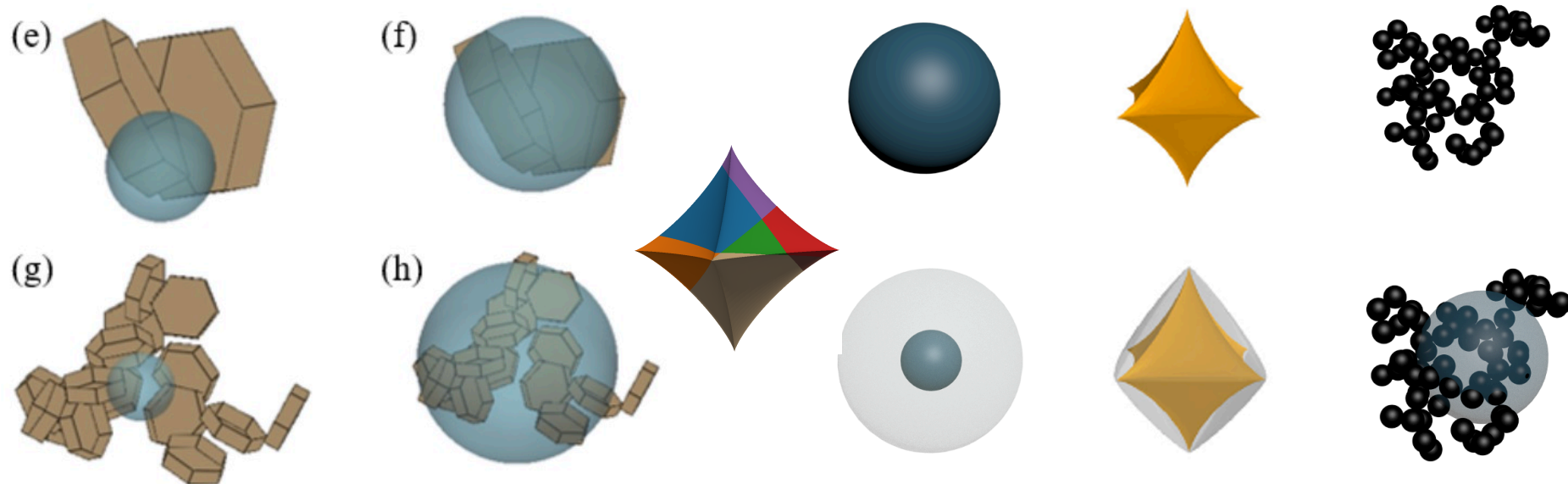
# G-IITM: A GPU Implementation



The computation speed increases by **10~20 times**.

# Level of particle complexity is significantly improved.

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# IITM T-matrix Features

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- Arbitrary shaped and inhomogeneous particles (**flexibility**)
- Analytical random orientation average (**accuracy and efficiency**)
- Particle size parameter up to geometric-optics domains (**applicability**)

Conventional EBCM T-matrix: homogeneous axially symmetric particles.

# Progresses

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- **T-matrix, Super-formula, Machine Learning**

# Freedom

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**Sphere: No shape freedom**

**Spheroid: one**

**Tri-axial ellipsoids: two**

**Super-formula: more freedom**

**Atmospheric particles: infinite, optimized freedom?**

# Why more freedom is useful?

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## Drawing an elephant with four complex parameters

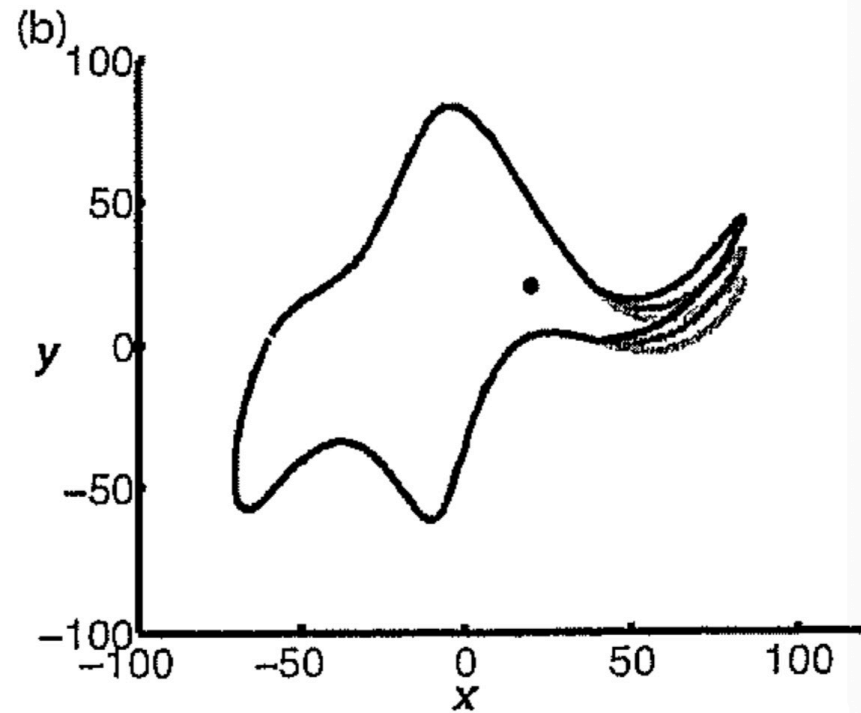
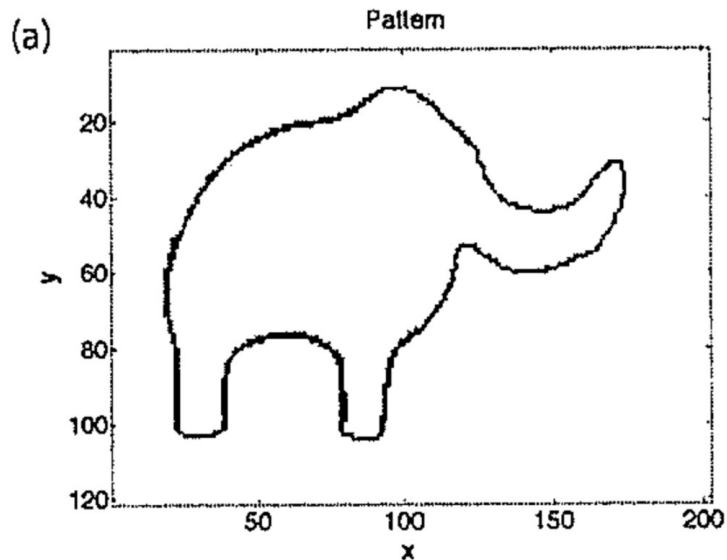
Jürgen Mayer  
*Max Planck Institute of Molecular Cell Biology and Genetics, Pfotenhauerstr. 108, 01307 Dresden, Germany*

Khaled Khairy  
*European Molecular Biology Laboratory, Meyerhofstraße. 1, 69117 Heidelberg, Germany*

Jonathon Howard  
*Max Planck Institute of Molecular Cell Biology and Genetics, Pfotenhauerstr. 108, 01307 Dresden, Germany*

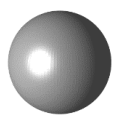
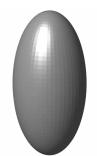

(Received 20 August 2008; accepted 5 October 2009)

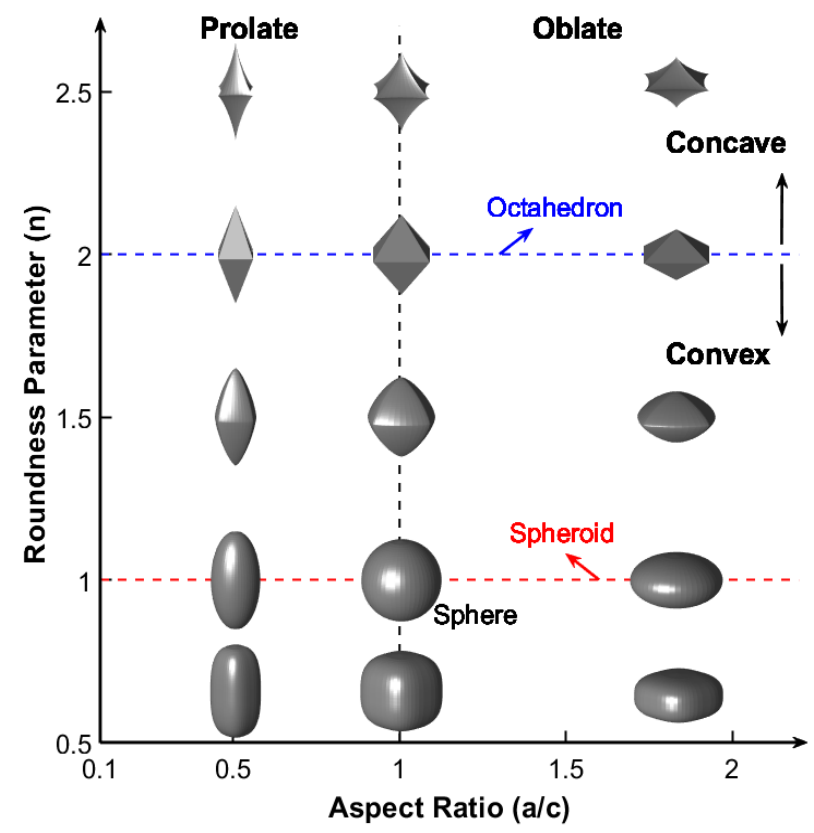
We define four complex numbers representing the parameters needed to specify an elephantine shape. The real and imaginary parts of these complex numbers are the coefficients of a Fourier coordinate expansion, a powerful tool for reducing the data required to define shapes. © 2010 American Association of Physics Teachers.  
[DOI: 10.1119/1.3254017]



Wiggling Trunk

# Super-spheroid model

 **sphere**  $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 + \left(\frac{z}{a}\right)^2 = 1$   
 **spheroid**  $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$   
**super-spheroid**  $\left(\frac{x}{a}\right)^{2/n} + \left(\frac{y}{a}\right)^{2/n} + \left(\frac{z}{c}\right)^{2/n} = 1$   
  $n=0.15$   
**aspect ratio:  $a/c$**   
**roundness parameter:  $n$**

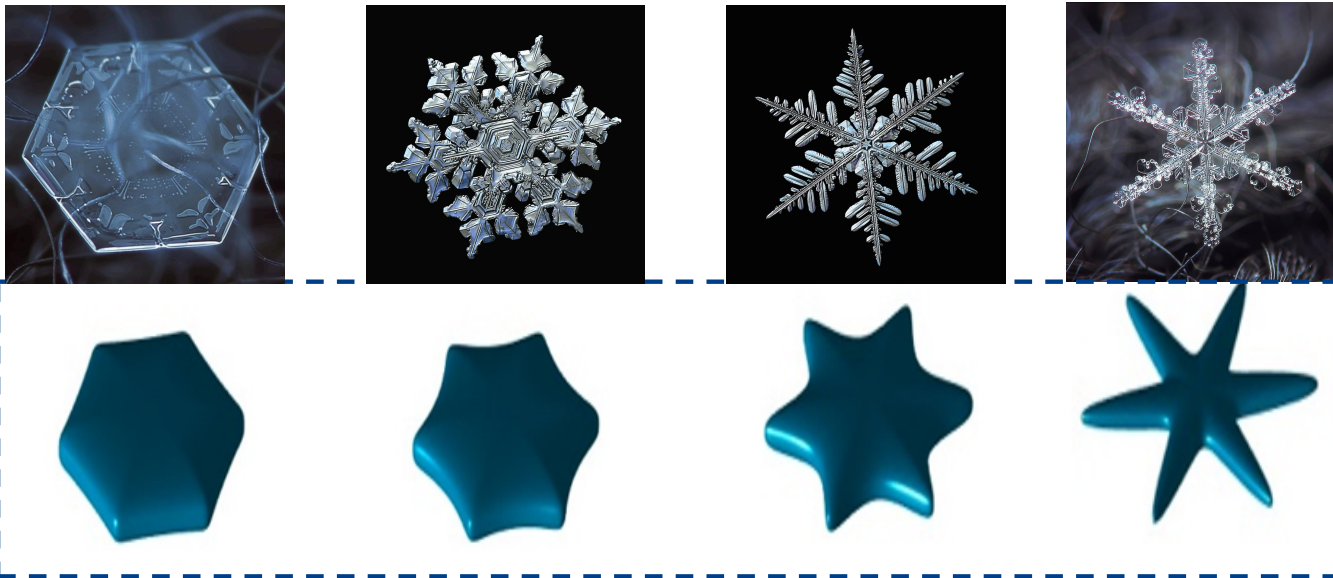




# Continuous Parameter Snowflake Model

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Picture from  
alexey\_kljatov



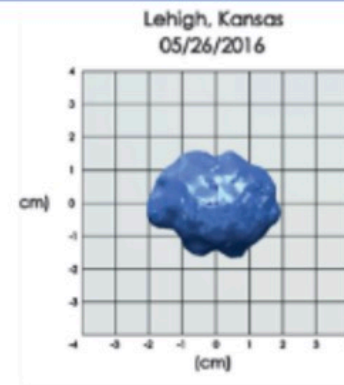
**Superformula** 
$$r(\phi) = a \left[ 2^{\frac{n_2}{2}-1} \left( |\cos(1.5\phi)|^{n_2} + |\sin(1.5\phi)|^{n_2} \right) \right]^{-\frac{1}{2n_1}}$$

➤ **Shape parameter  $n_1$ ,  $n_2$  controls continuous change of snowflake shape**

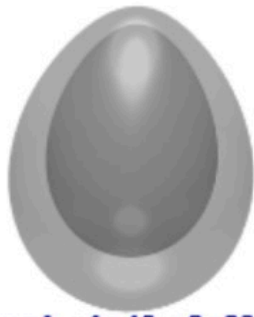
# Graupel

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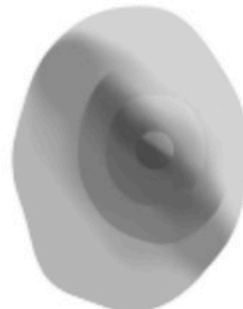
**Observations**



**Formula**



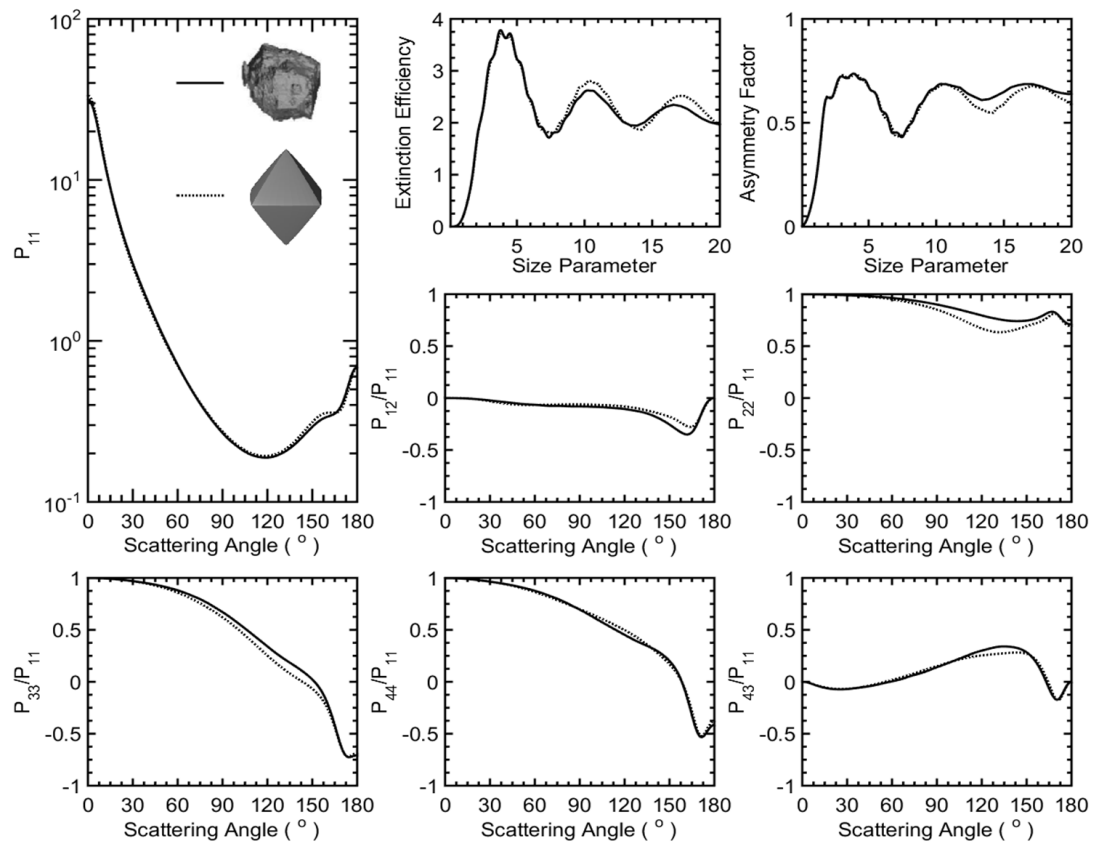
(b)



(c)

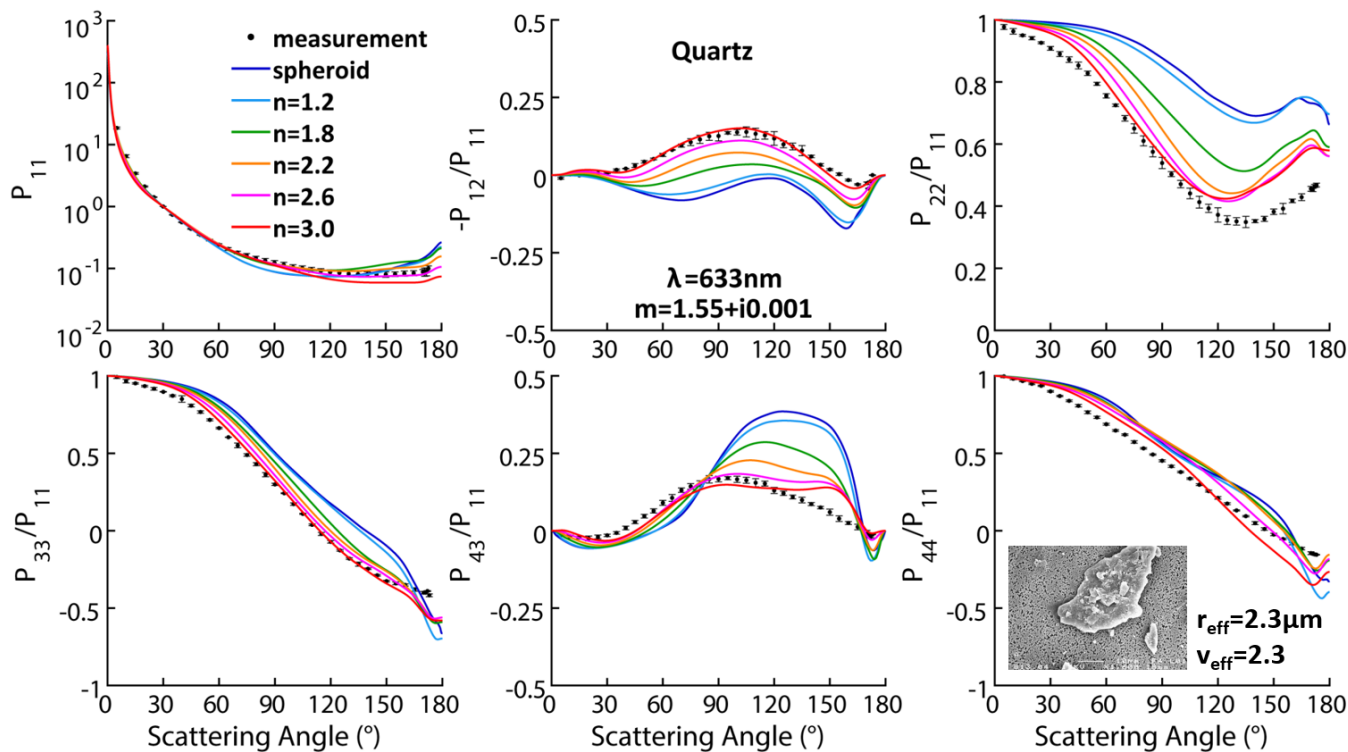


# Optical “equivalence”

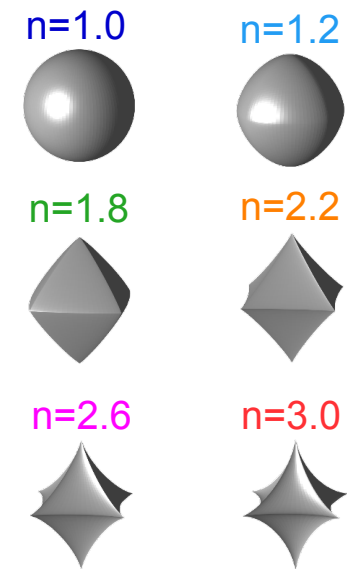


# Modeling scattering matrices of dust aerosols

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Equi-probable aspect ratio distribution

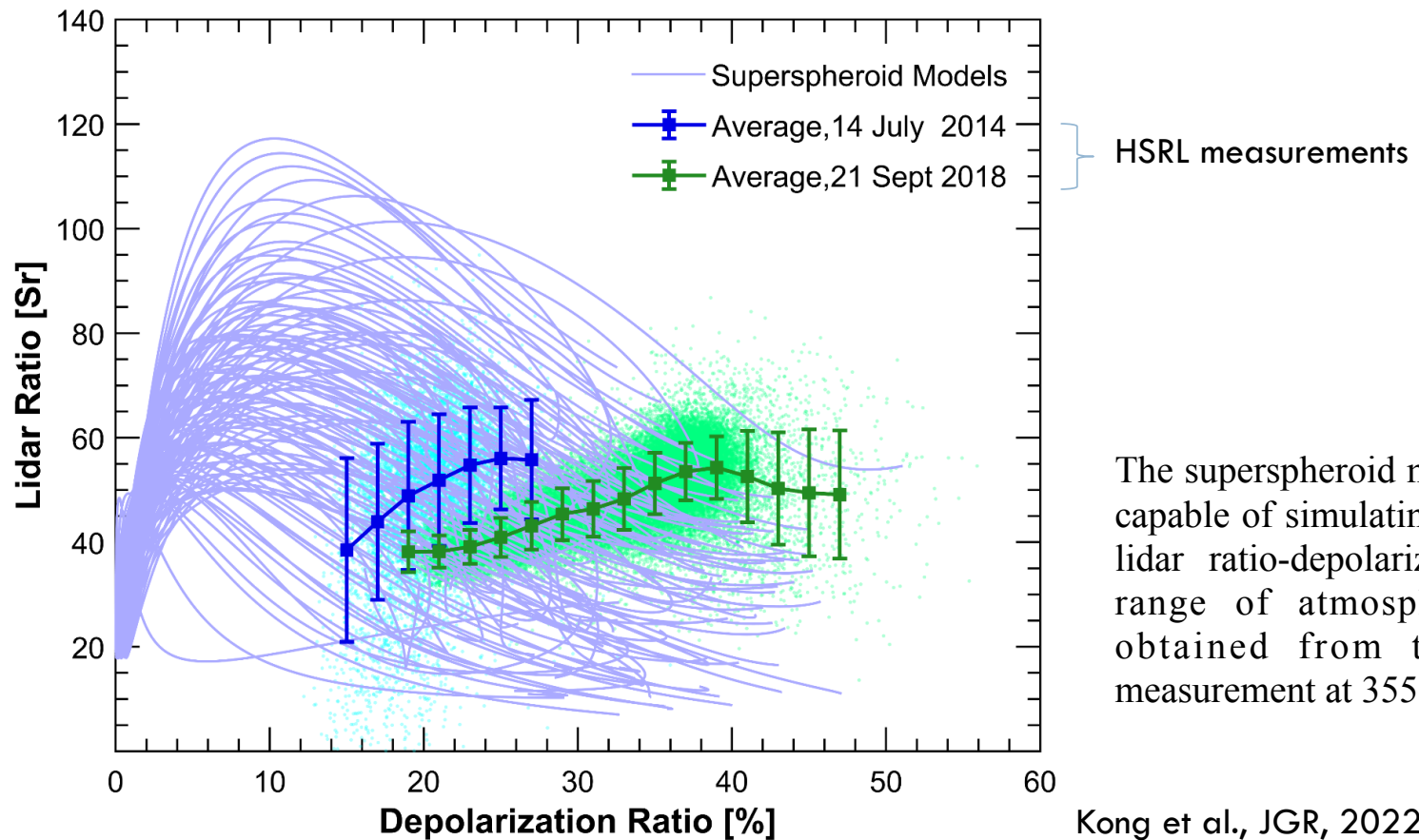


Lin et al., *JGR*, 2018

- Concave super-spheroids can reproduce the scattering matrices of dust samples from the Amsterdam-Granada Light Scattering Database.

# Modeling the depolarization ratio and lidar ratio of dust

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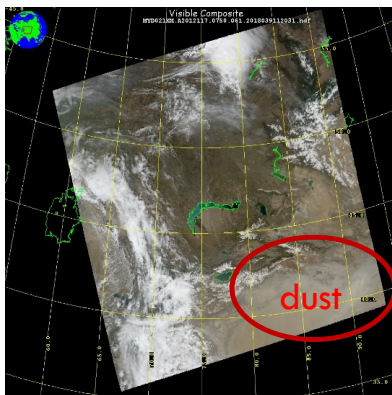
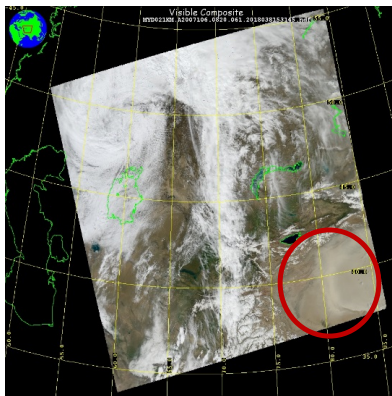


The superspheroid models were capable of simulating the entire lidar ratio-depolarization ratio range of atmospheric dust obtained from the HSRL measurement at 355 nm.

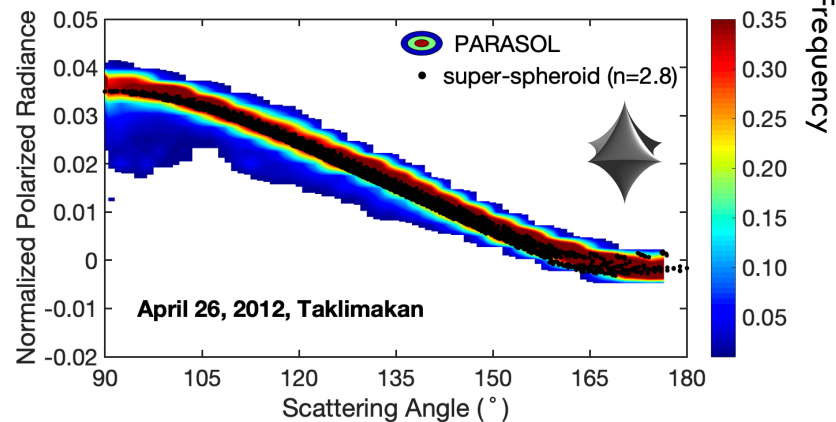
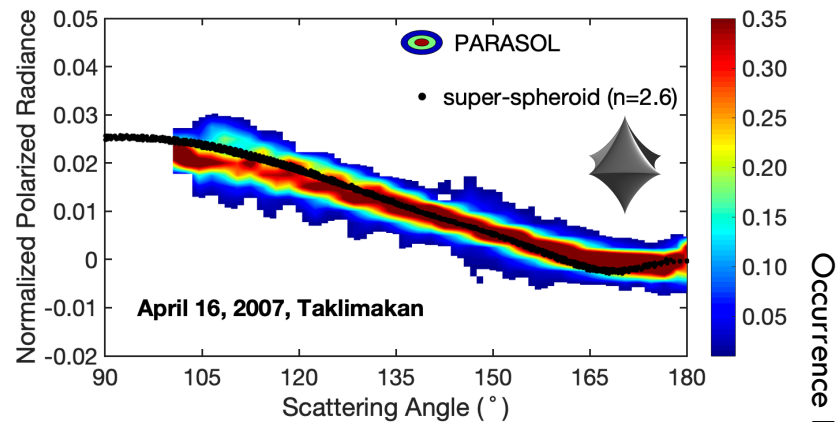
Kong et al., JGR, 2022

# Modeling PARASOL observations under dusty-sky conditions

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From MODIS

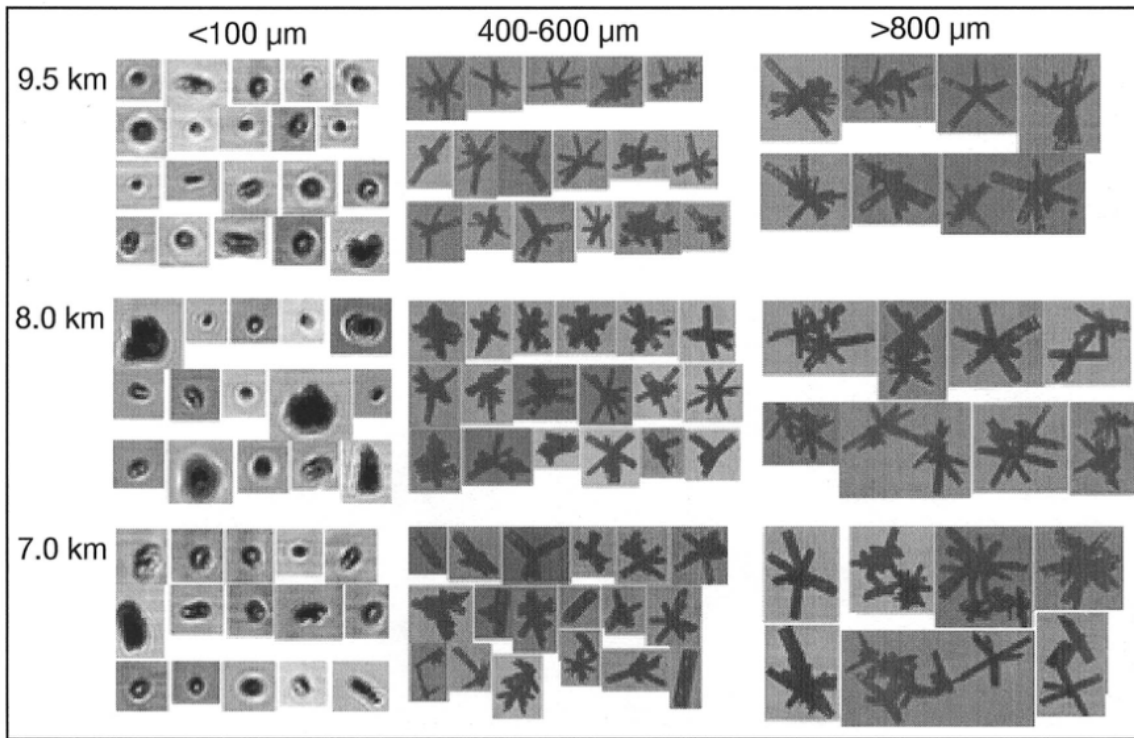


- Concave super-spheroids successfully reproduce the angular distribution of the observed TOA polarized radiance under dusty-sky conditions.

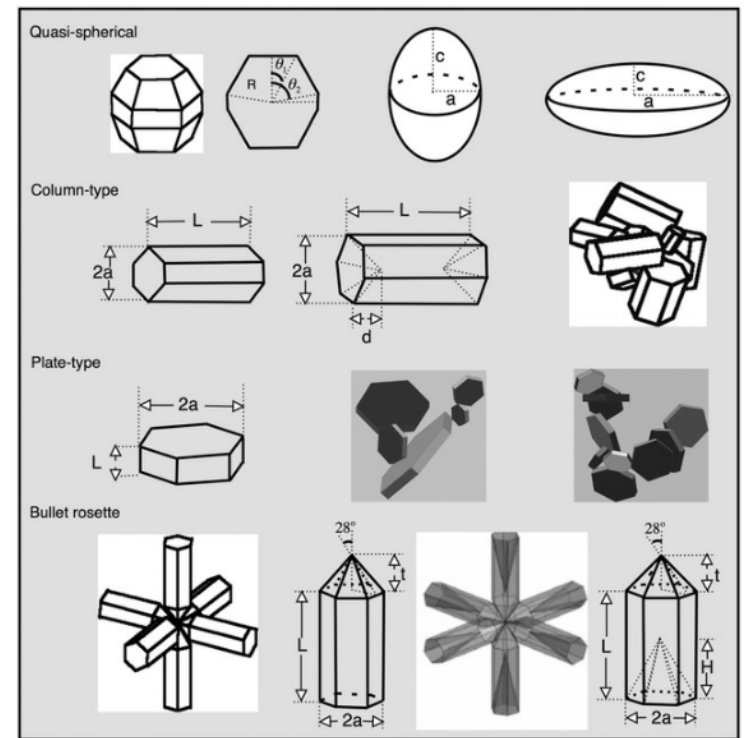
Lin et al., JGR, 2021

# Different shapes and physical models of ice crystal

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Heymsfield, et al., 2003

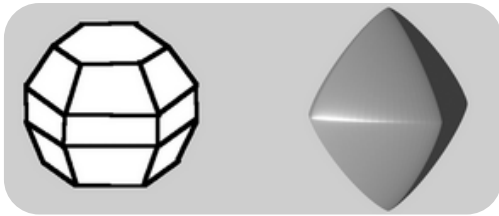


Yang, Bi, et al., 2013

- Electromagnetic wave scattering by ice particles is commonly modeled by defining representative habits, including droxtals, columns, plates, and aggregates, although actual particles in the atmosphere can be even much more complex.

# The use of super-spheroids as surrogates for ice crystal

Droxtal Super-spheroid



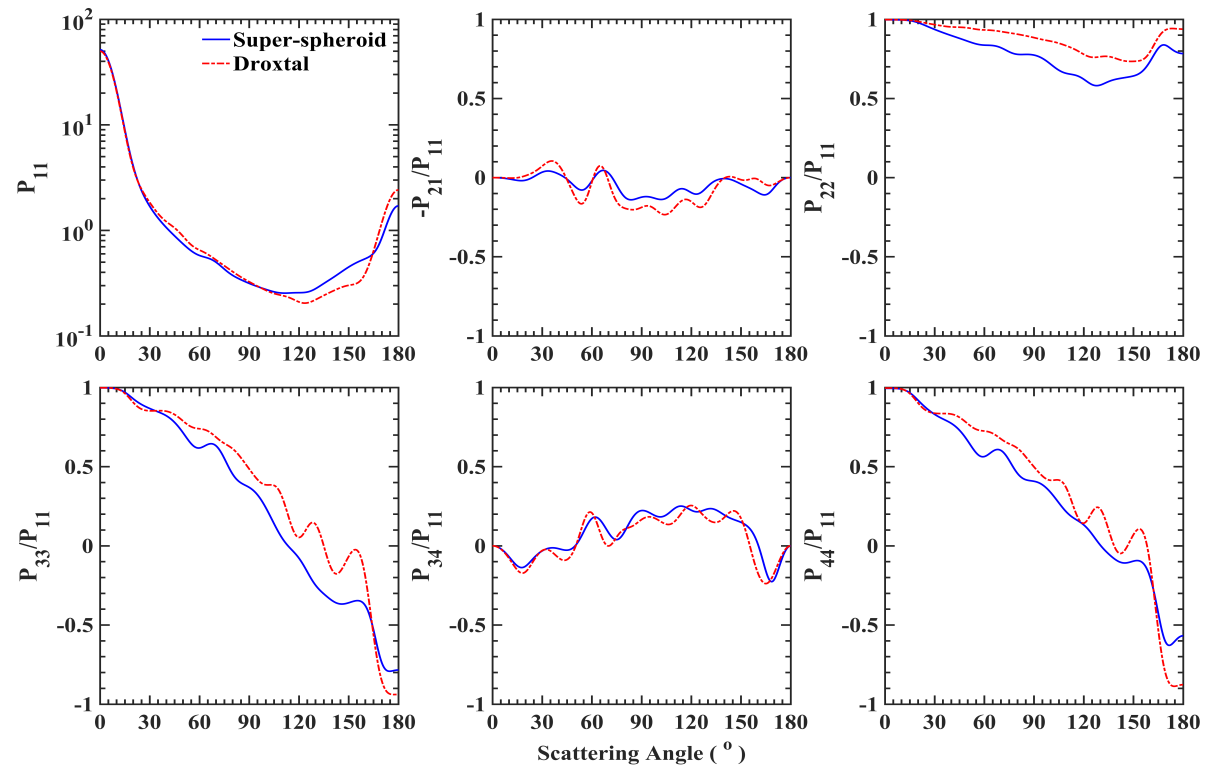
Optical “equivalence”

$$SI = \frac{3V}{4\pi S/\pi^{3/2}}$$

V = Volume

S = Average projected area

## Scattering Matrix

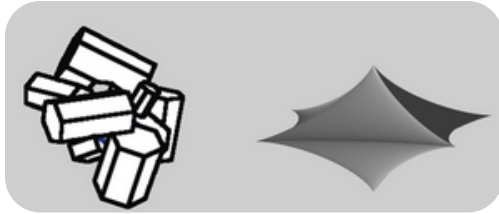


Sun et al., Remote Sensing, 2021



# The use of super-spheroids as surrogates for ice crystal

Aggregate      Super-spheroid



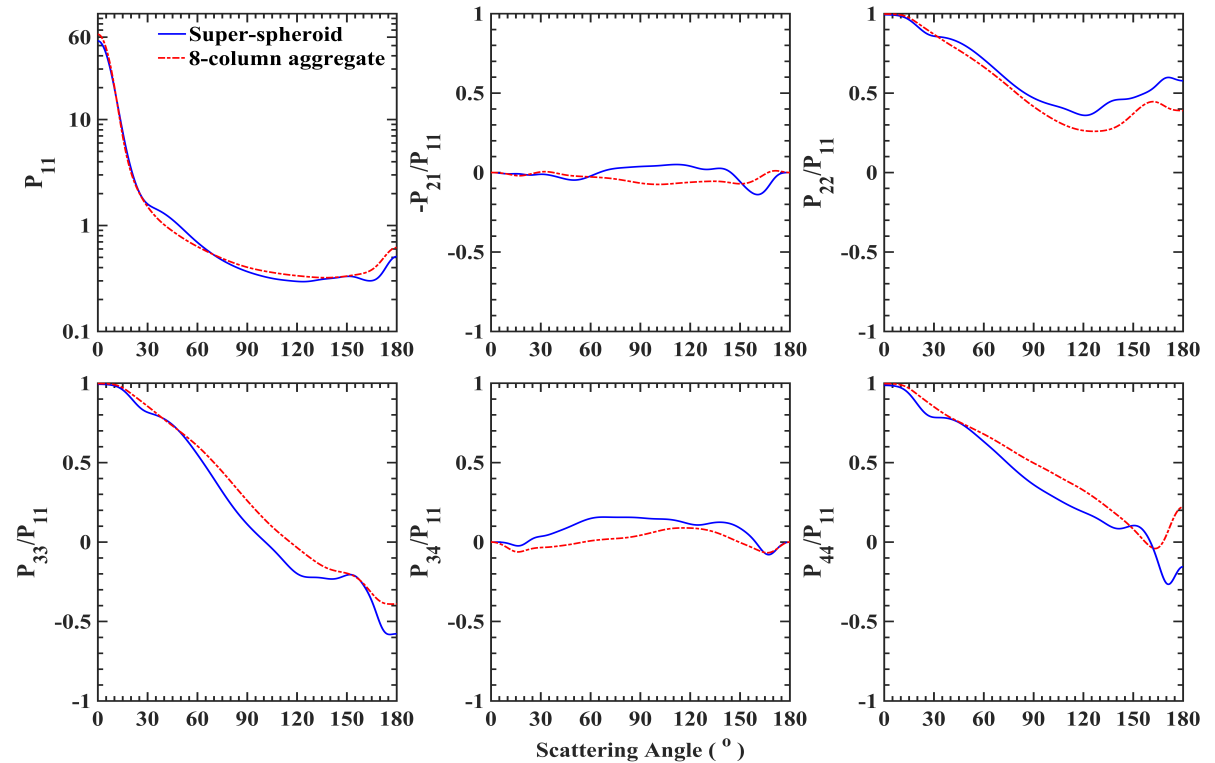
Optical “equivalence”

$$SI = \frac{3V}{4\pi S/\pi^{3/2}}$$

V = Volume

S = Average projected area

Scattering Matrix



Sun et al., Remote Sensing, 2021

# Progresses

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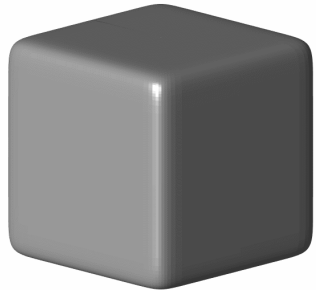
- **T-matrix, Super-formula, Machine Learning**

# Super-spheroids Scattering Database

## Super-spheroid

$$\left(\frac{x}{a}\right)^{2/n} + \left(\frac{y}{a}\right)^{2/n} + \left(\frac{z}{c}\right)^{2/n} = 1$$

**n=0.15**

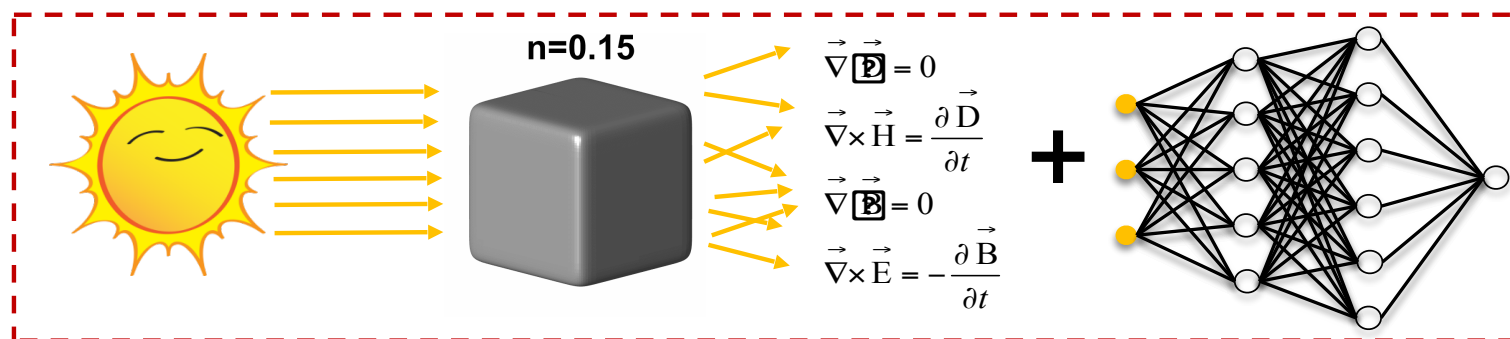


| <u>Aspect ratio</u> | <u>Roundness</u> | <u>Refractive index</u> |                     | <u>Size parameter</u> | <u>Scattering angle (°)</u> |
|---------------------|------------------|-------------------------|---------------------|-----------------------|-----------------------------|
|                     |                  | <u>Real part</u>        | <u>+ Imag. part</u> |                       |                             |
| 0.5                 | 1.2              | 1.30                    | 10 <sup>-7</sup>    | 0.1                   | 0                           |
| 0.6                 | 1.4              | 1.35                    | 10 <sup>-6</sup>    | 0.2                   | 0.25                        |
| 0.7                 | 1.6              | 1.40                    | 10 <sup>-5</sup>    | .....                 | 0.5                         |
| 0.8                 | 1.8              | 1.45                    | 10 <sup>-4</sup>    | 9.9                   | 0.75                        |
| 0.9                 | 2.0              | 1.50                    | 0.001               | 10                    | 1                           |
| 1.0                 | 2.2              | 1.55                    | 0.005               | 10.2                  | .....                       |
| 1.2                 | 2.4              | 1.60                    | 0.01                | .....                 | 179                         |
| 1.4                 | 2.6              | 1.65                    | 0.05                | 19.8                  | 179.25                      |
| 1.6                 | 2.8              | 1.70                    | 0.1                 | 20                    | 179.5                       |
| 1.8                 | 3.0              | 1.75                    |                     | 21                    | 179.75                      |
| 2.0                 |                  | 1.80                    |                     | .....                 | 180                         |
|                     |                  |                         |                     | 49                    |                             |
|                     |                  |                         |                     | 50                    |                             |

- The database is too **huge (~127GB)** to share and apply

# Deep learning on the Super-spheroids Scattering Database

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**Input**

super-spheroids  
particle parameters

- Aspect ratio
- Roundness
- Ref. index Real part
- Ref. index Imag. part
- Size parameter
- Scattering angle

**DNN model**

**Output**

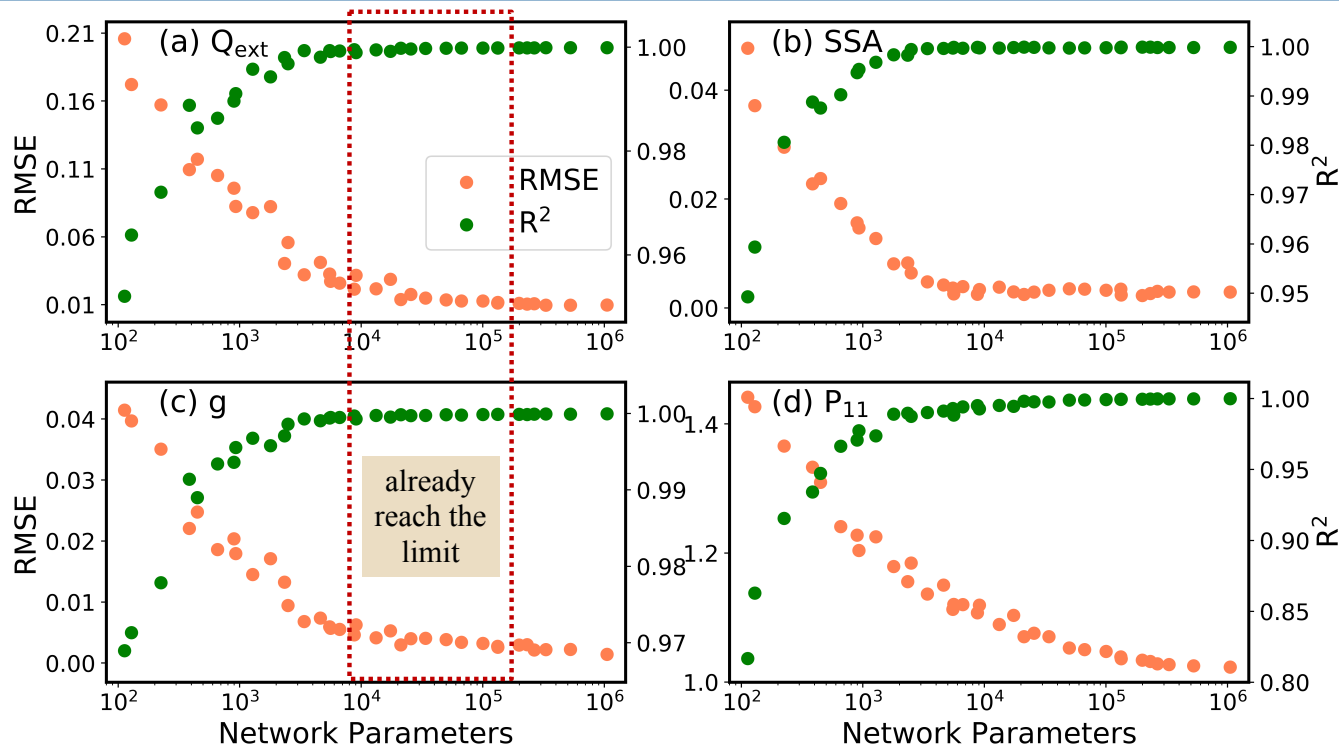
optical properties

- Extinction efficiency factor
- Single scattering albedo
- Asymmetry factor
- Phase matrix elements

• **Note: Shapes are training parameters.**

# Choose appropriate models via RMSE-R<sup>2</sup>-Network parameters

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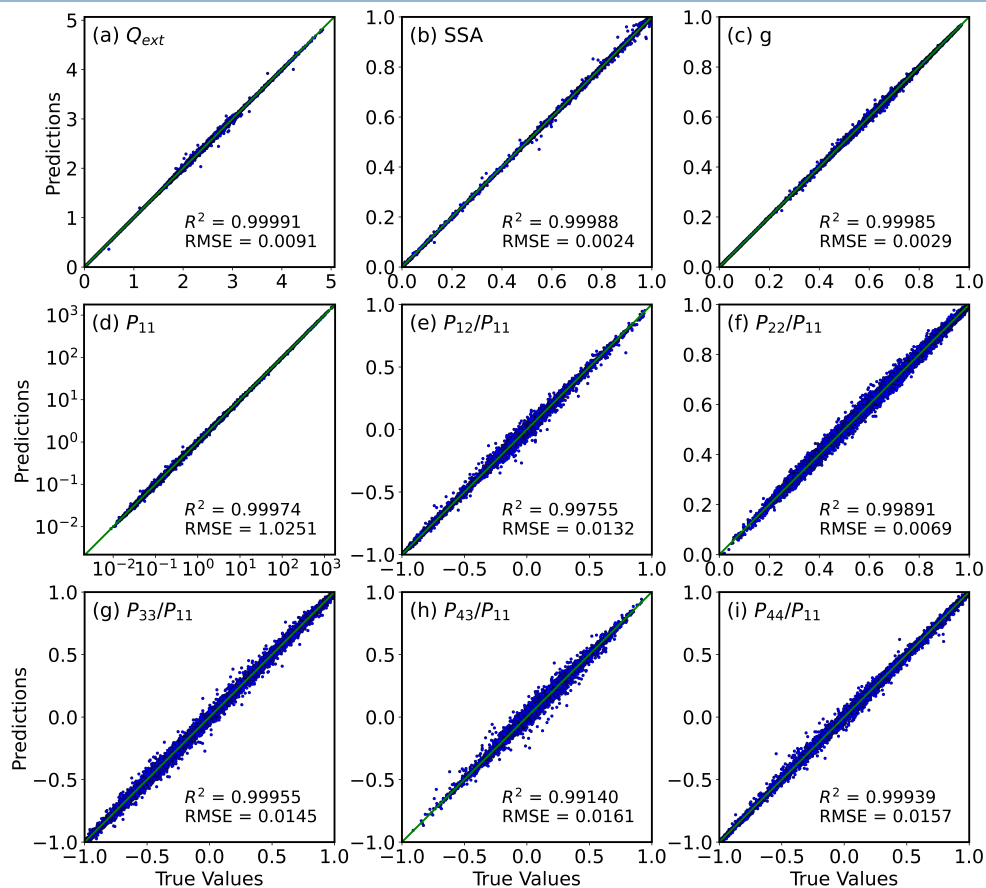


Small size ( $\sim 10^4$  parameters inside) DNN models can gain low error and good performance, **replace the big database by  $\sim 6800$  compression ratio.**

J. Yu, Lei Bi, Wei Han, Xiaoye Zhang, Advances in Atmospheric Sciences, 2022

# Performance of the optimal models

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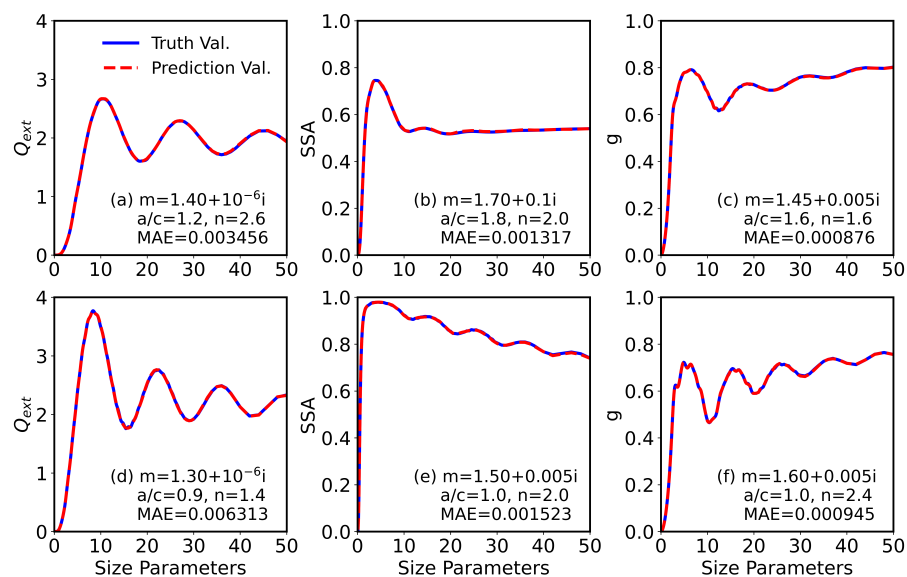
- Performance on the test set (including 100,000 samples)

$R^2 > 0.99$

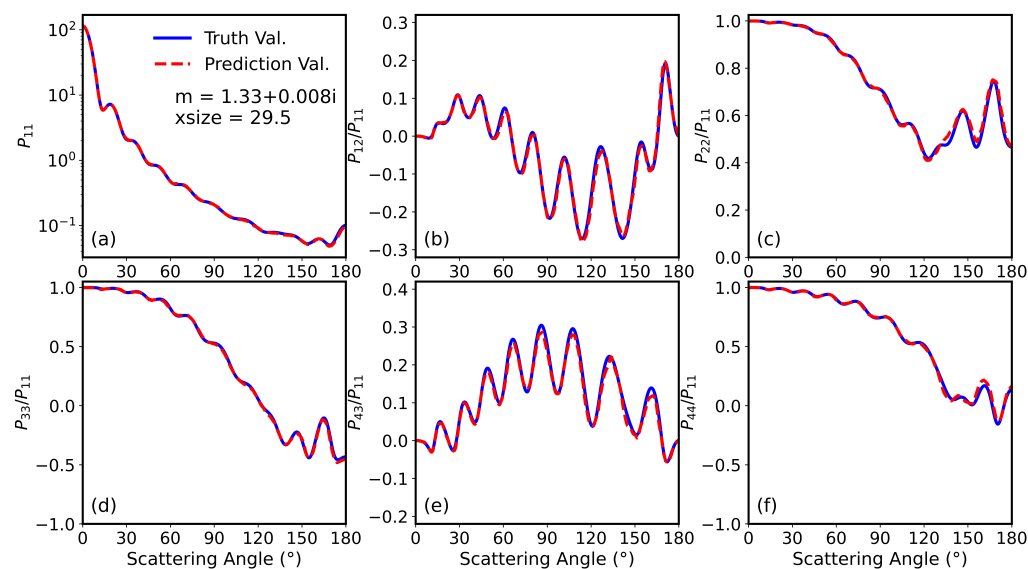
# Deep learning on the Super-spheroids Scattering Database

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## Test on specific known particles



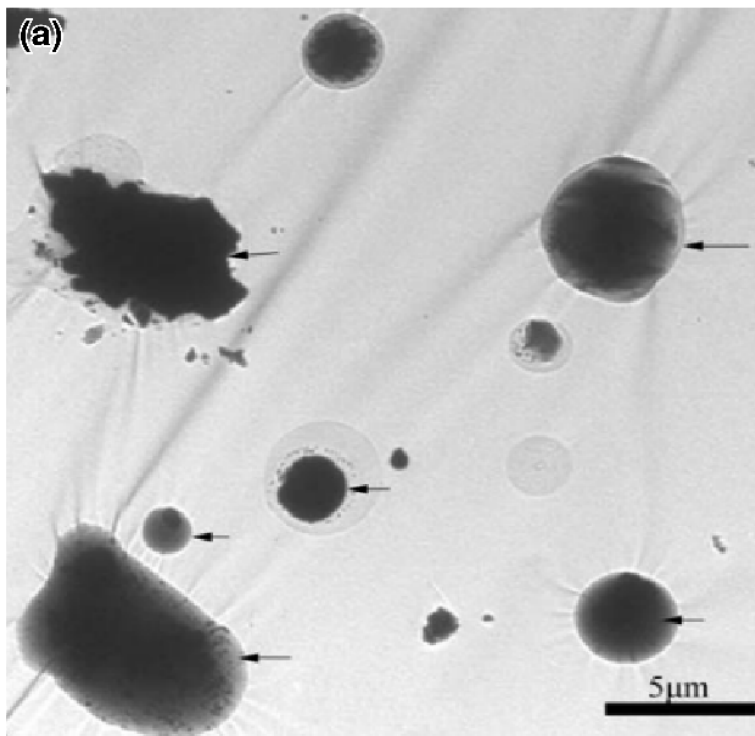
## Test on specific known particles



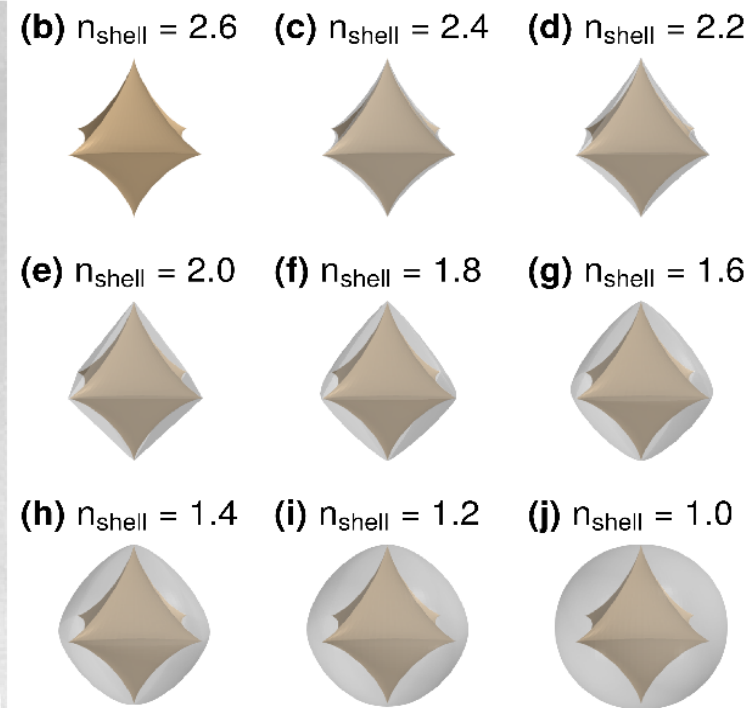
- **The DNN models can accurately predict the optical properties of specific particles which are already existed in the database or unknown before.**

# A new optical scheme for inhomogeneous particles

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(Li and Shao, 2009)

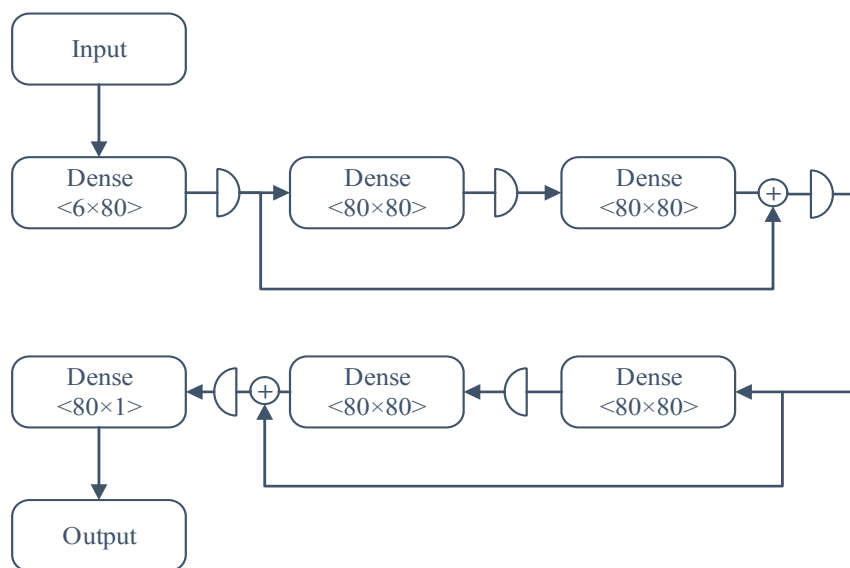


(Wang et al., JQSRT, 2022)



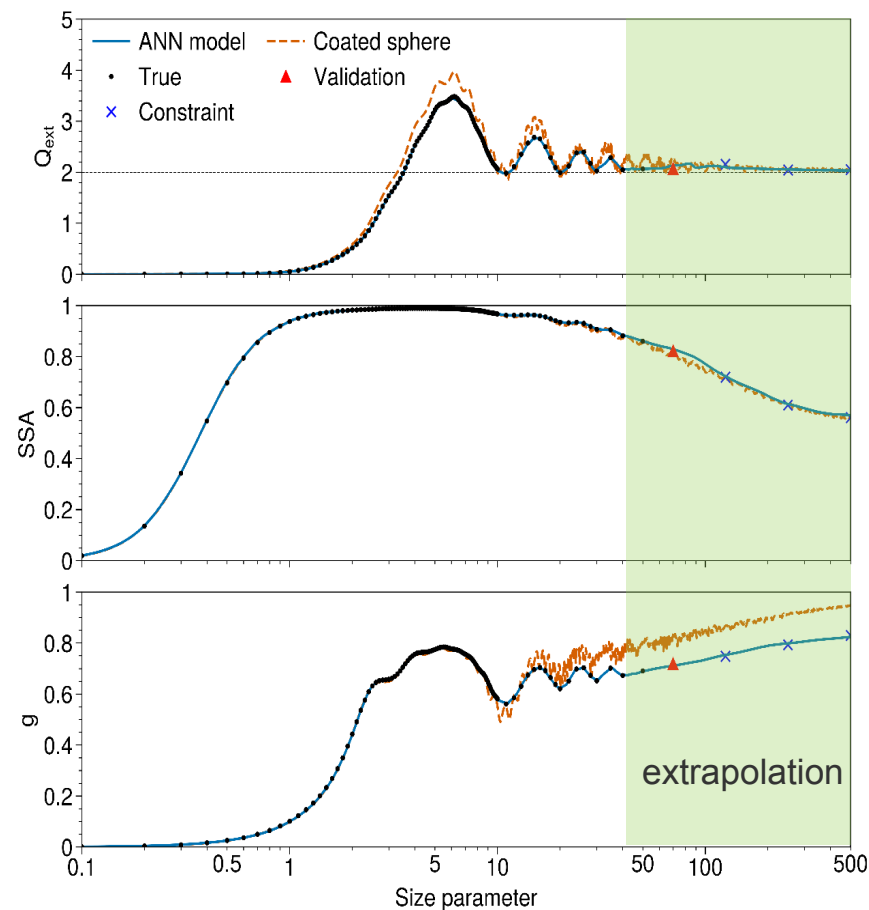
# Use of residual networks

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Design of the ResNet

The ResNet is used to predict the optical properties of large-size particles.



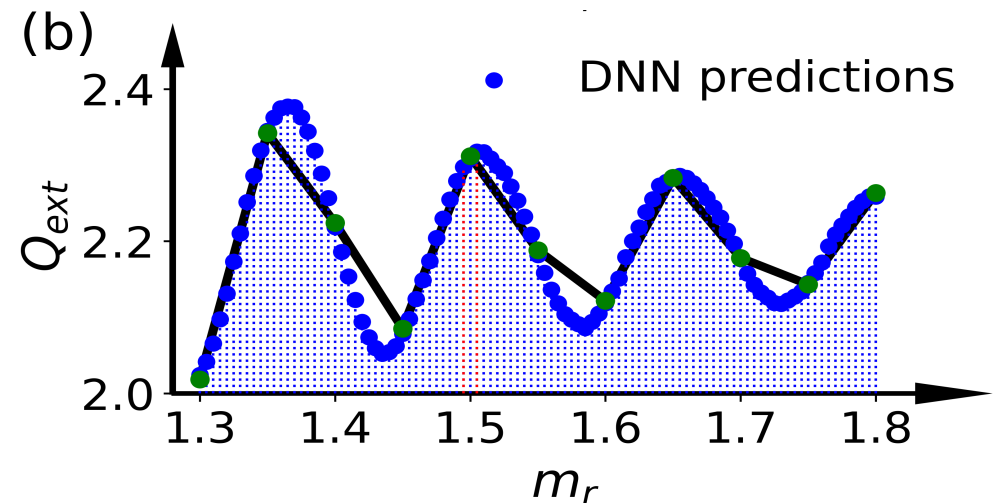
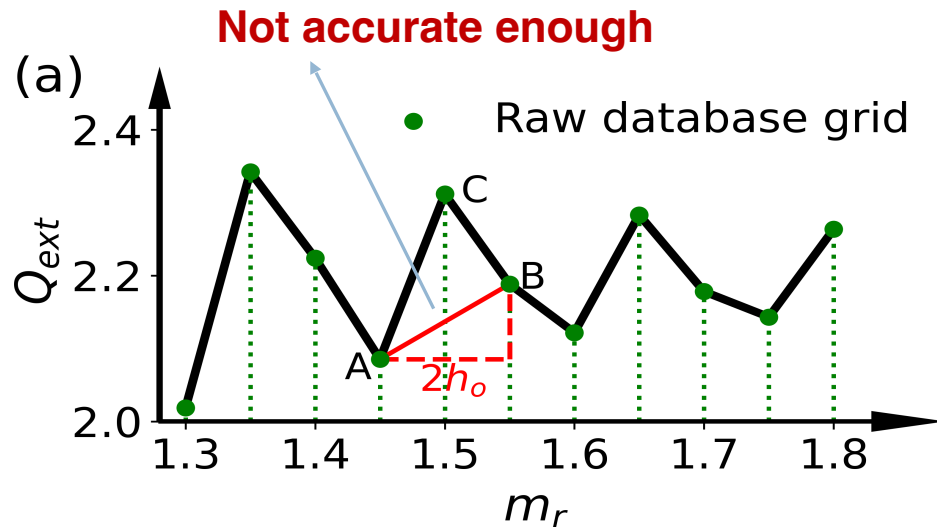
## Optical property Jacobians

- Derivatives of extinction efficiency, single scattering albedos, scattering phase matrix respect to shape parameters, refractive indices (real and imaginary part), and size parameters.

# Jacobians of optical properties computed using neural networks

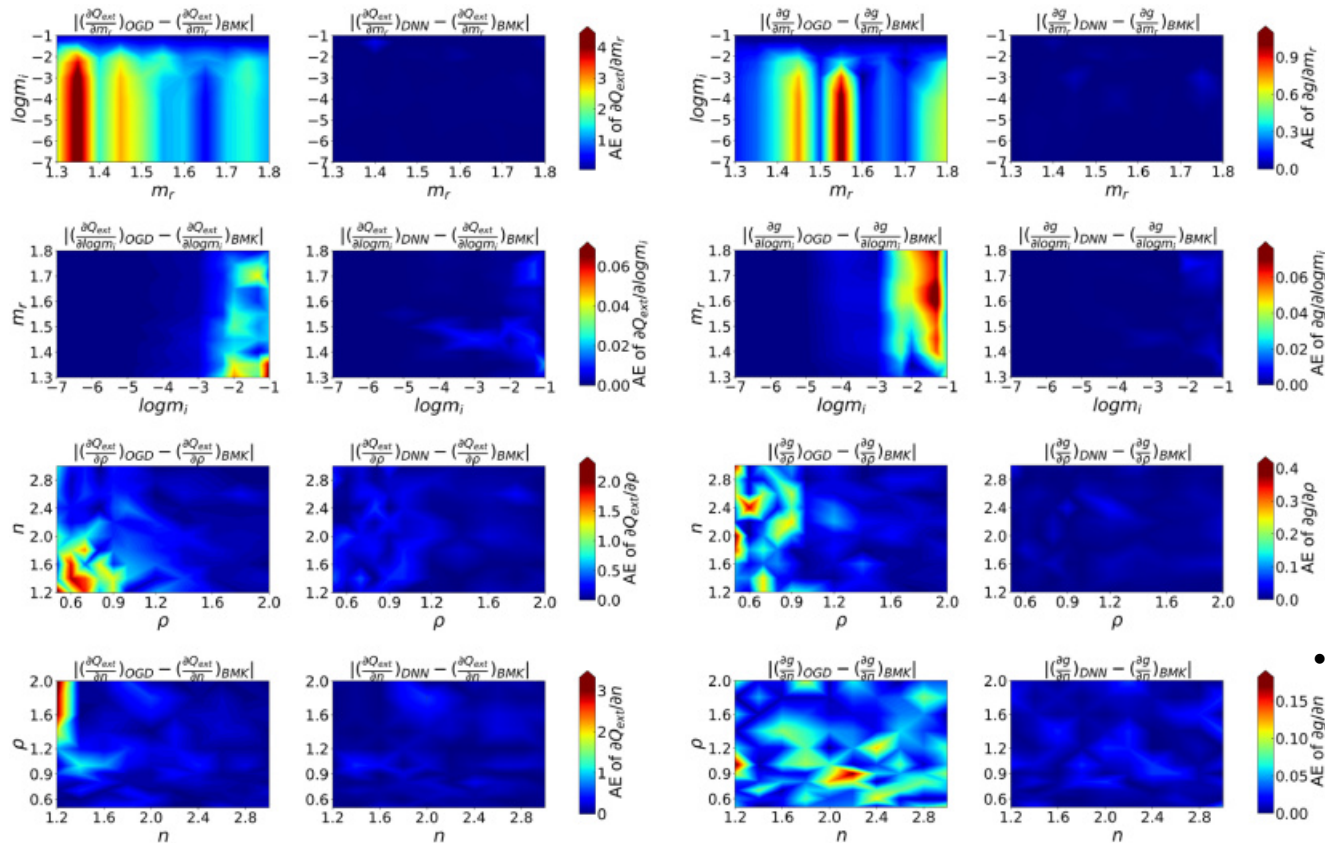
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## Finite Difference Method (FDM)



- The neural networks provide a large number of grid points, so that the FDM grid interval becomes denser and more accurate partial derivatives are calculated.

# Comparison of with Benchmarks



Compare the absolute error of the raw grid point and NN scheme with the benchmark

The mean absolute errors are reduced by more than **60%**.

• Yu et al., Optics Express, 2022

## Benefits of machine learning

- No repeat light scattering calculations when particle models, refractive indices and size parameters are updated.
- Significantly compress the original optical property database.
- Look-up tables can be conveniently replaced with neural networks.

# Applications +++

# Geophysical Research Letters

Research Letter |  Open Access |    

## More or Less: How Do Inhomogeneous Sea-Salt Aerosols Affect the Precipitation of Landfalling Tropical Cyclones?

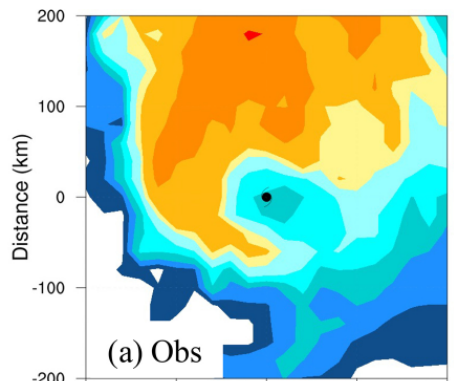
Limin Zhu, Shoujuan Shu , Zheng Wang, Lei Bi 

First published: 28 January 2022 | <https://doi.org/10.1029/2021GL097023>

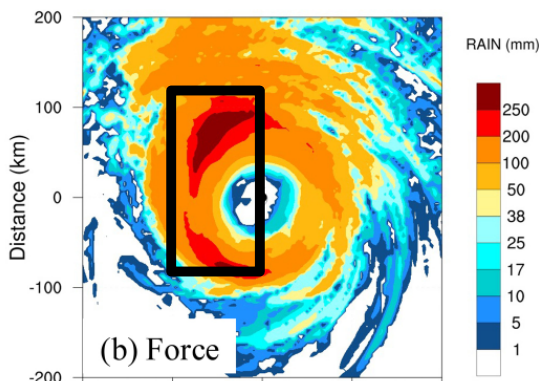
# The inhomogeneity of sea salt further suppresses the rainfall.

## Observation and simulation of rainfall of Fitow

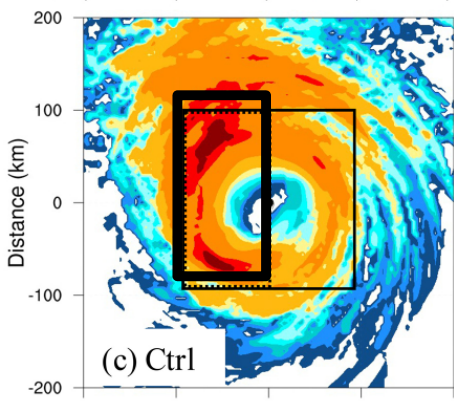
Observation



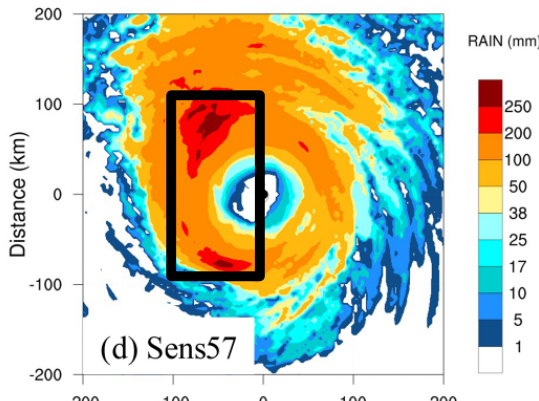
Without sea slat



Homogeneous



Inhomogeneous ✓



- The inhomogeneity of sea salt suppresses the rainfall.







# Journal of Quantitative Spectroscopy and Radiative Transfer

Volume 283, June 2022, 108147

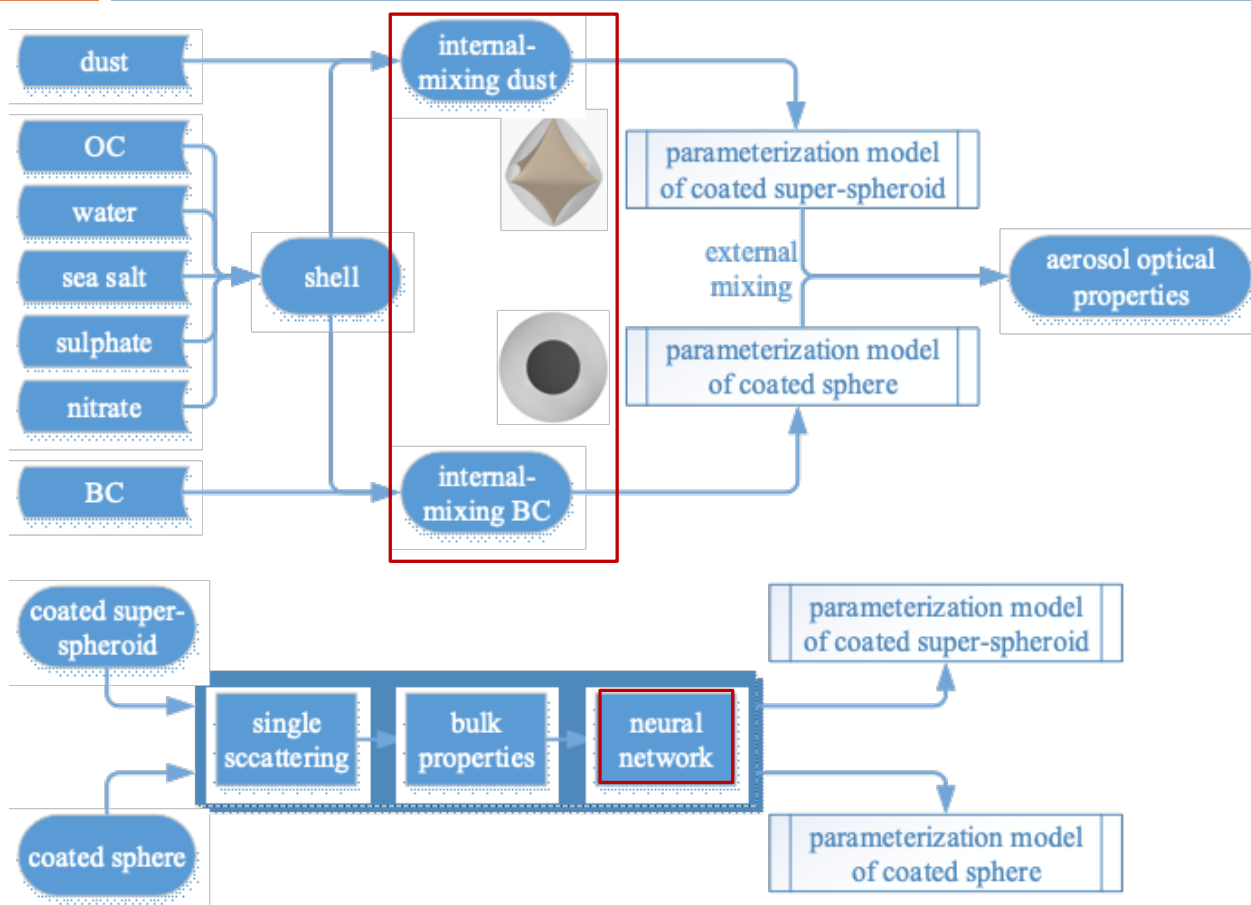


## Evaluation of a new internally-mixed aerosol optics scheme in the weather research and forecasting model

Zheng Wang<sup>a</sup>, Lei Bi<sup>a</sup>  , Hong Wang<sup>b</sup>, Yaqiang Wang<sup>b</sup>, Wei Han<sup>c</sup>, Xiaoye Zhang<sup>b</sup>

# A new internally-mixed aerosol optical scheme

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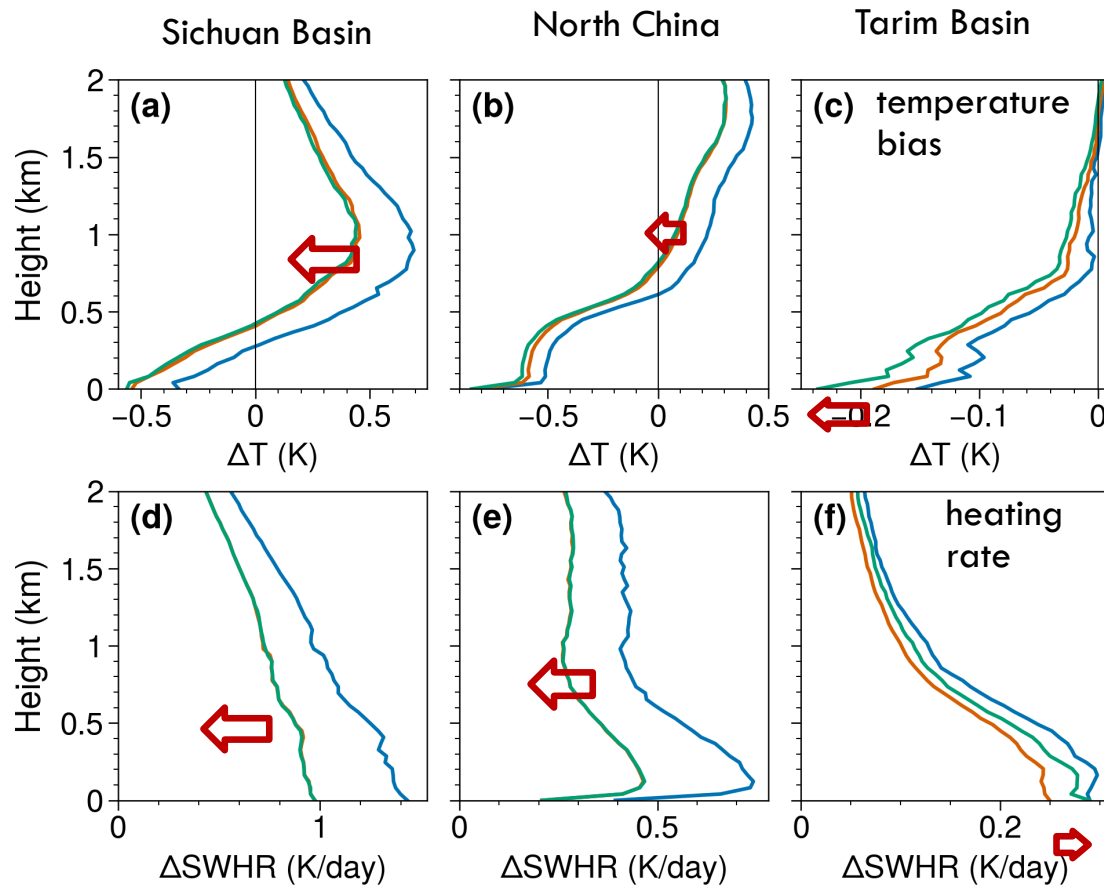


- **Inhomogeneity of the internal mixing**
- **Non-sphericity of dust-containing particles**
- **Parameterization by neural network**

Wang et al., JQSRT, 2022

# Implemented in the weather research and forecasting model

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## Effect of nonsphericity and inhomogeneity on the boundary layer

— Sphere  
— Coated sphere  
— New scheme

- The inhomogeneity of aerosols reduces the solar heating effect.
- The nonsphericity of aerosols enhances the surface dimming effect.

Wang et al., JQSRT, 2022

# Summary

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**Key words:** T-matrix, Super-formula, Machine Learning

**We DONOT** hope that all particles are spheres,  
because nonspherical particles are so fun.

# Hangzhou, China





浙江大學  
ZHEJIANG UNIVERSITY

**THANKS!**