Estimation of AMSU –A radiance error statistics, using an optimality criterion

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Definition of the tuning

Denoting ε_b and ε_o the background and observational errors, let $\mathbf{B}$ be the background error covariance matrix specified in an operational data assimilation system and let $\mathbf{B_t} = E(\varepsilon_b.\varepsilon_b{}^T)$ be the "true" matrix (E is the mathematical expectation operator). In the same way let us define $\mathbf{R}$ and $\mathbf{R_t}$ « true » $= E(\varepsilon_O.\varepsilon_O{}^T)$ as the specified and true observational error covariance matrices, and $\mathbf{R_k}$, $\mathbf{R_{tk}}$ and $\mathbf{B_l}$, $\mathbf{B_{tl}}$ the specified and true matrices related to the kth type of observations and to the lth independent subpart of the control vector.

Assuming that one can write

$$\mathbf{B_t} = s_b\,\mathbf{B},\ \mathbf{R_t} = s_o\,\mathbf{R}$$

or

$$\mathbf{R_{tk}} = s_{ok}\,\mathbf{R_k},\ \mathbf{B_{tl}} = s_{bl}\,\mathbf{B_l},$$

the aim of this work is to answer the question: how to evaluate the tuning coefficients: s_o and s_b (s_{ok} and s_{bl})?

Desroziers and Ivanov (2001) proposed to use an optimality criterion found by Talagrand (1999). The tuning coefficients are those for which this criterion if fulfilled.

Desroziers and Ivanov's method (2001)

Let $J = \Sigma J_{ok} + \Sigma J_{bl}$ be the suboptimal cost function used in an operational system; let $J_t = \Sigma J_{ok}/s_{ok} + \Sigma J_{bl}/s_{bl}$ be the optimal cost function using « true » matrices.

If x_a is the minimizer of J_t then, following Talagrand (1999), the expectations of the subparts of the cost function at the minimum are:

$$E(2J_{ok}(x_a)/s_{ok}) = Tr\,[\pi_k(I_p - HK)\pi_k{}^T]$$
$$E(2J_{bl}(x_a)/s_{bl}) = Tr\,(\pi_l KH\pi_l{}^T).$$

Where $\mathbf{K}$ is the Kalman gain matrix, $\mathbf{H}$ is the observation operator, and π_k and π_l are respectively the projections onto the kth type of observations and to the lth independent subpart of the control vector. The latter equations may be written

$$s_{ok} = 2J_{ok}(x_a)/\,Tr\,[\pi_k(I_p - HK)\pi_k{}^T]$$
$$s_{bl} = 2J_{bl}(x_a)/\,Tr\,(\pi_l KH\pi_l{}^T).$$

In this case $\mathbf{K}$ can be written as a function of s_{ok} and s_{bl}. This is therefore a fixed-point relation. We will use a fixed-point algorithm to compute the tuning coefficients, going from step i to step $i+1$ using the following relations:

$$s^{(i+1)}{}_{ok} = 2J_{ok}(x_a\,(s^{(i)}))/\,Tr\,[\pi_k(I_p - HK^{(i)})\pi_k{}^T],\ \forall k$$
$$s^{(i+1)}{}_{bl} = 2J_{bl}(x_a(s^{(i)}))/\,Tr\,(\pi_l K^{(i)}H\pi_l{}^T),\ \forall l.$$

It is to be noticed that no matrix is explicitly stored in the system. In order to compute Tr (HK) one can use a randomized trace estimation technique, several of them exist. For example, Desroziers and Ivanov adapted a method proposed by Girard (1987).

Some properties of the method

Chapnik *et al*. (2003) have shown that:

➤the method is equivalent to a Maximum likelihood tuning of the variances. Maximum likelihood is a general method to tune parameterized probability densities with respect to observed data; its application in data assimilation is discussed in Dee and da Silva (1998);

➤the quality of the estimates depend on the number of observations;

➤tuning independently the guess errors and the observation errors or two statistically independent observation error variances has little or no effect on the coefficients;

➤Observations with spatially correlated errors, analyzed with a prescribed diagonal R matrix may have their variance (possibly grossly) under estimated by the method;

➤the computed values are temporally stable (up to four years); on the contrary they react quickly and increase when the quality of observations is degraded: they behave like variances are supposed to.

Moreover, as already stated by Desroziers and Ivanov, the first iteration of the fixed-point algorithm is a good approximation of the final result. It is possible to show that it is a biased estimate of the tuning coefficients, the more observations there are, the smaller the bias is.

Another implementation of the method

Following Sadiki and Fischer (2003), the fixed-point algorithm was used with one iteration only, and to increase the accuracy of the estimate, several situations were "concatenated":

If $J_{ok}^{i}(x_a)$ is the minimum of the subpart of the cost function related to observation type k on day i and Tr $[\pi_k(\mathbf{I}_p - \mathbf{H}^i\mathbf{K}^i)\pi_k^T]$ is its expected value if it were optimal, then the tuning coefficients are computed as

$$s_{ok} = (\Sigma_i \, J_{ok}^{i}(x_a)) \, / \, (\Sigma_i \, \mathrm{Tr} \, [\pi_k(\mathbf{I}_{pi} - \mathbf{H}^i\mathbf{K}^i)\pi_k^T] \,).$$

As suggested by Sadiki and Fischer, the different dates used in the computation are separated by at least 5 days in order to prevent time correlation.

Results

Results with simulated satellite radiances

Figure 1 shows the ability of the method to retrieve optimal variances in a simulated case. In this case the true standard deviations are the operational values and the mis-specified standard deviations are equal to the square root of the operational values; six dates, separated by more than five days, between 03/15/2003 and 05/19/2003 were used. Another computation was carried out with more thinning of the data in order to check the impact of a smaller number of observations. The standard deviations were computed for each of the three satellites NOAA15, NOAA16 and NOAA17, and a difference was made between sea and land observations. In all cases the computed deviations are fairly close to the expected ones.

Results with true observations

Estimation of TEMP temperature and wind speed vertical profile of standard deviations

The method was first tried with real observations, the standard deviations of which are supposed to be well documented: TEMP observations. Figure 2 shows the prescribed profile and the computed profile of standard deviations for wind speed. Figure 3 shows the same for temperature.

Those profiles were computed, cumulating the observations of 15 dates separated by five days between 03/15/2003 and 05/24//2003. The two figures clearly show that the computed deviations remain close to the prescribed ones, which was expected. In this case the estimates are realistic.

Estimation of AMSU A channels standard deviations.

With the same conventions as for Fig. 1, Fig. 4 shows the standard deviations computed for true data from 12 dates between 03/15/2003 and 05/24//2003. Some features appear: roughly, all the standard deviations are over estimated by a factor of 2. It can be seen that, according to this tuning, AMSU –A channel 5 has a larger standard deviation for land observations than for sea observations. Satellite NOAA16 instrument seems to have a larger standard deviation for channel 8 than the other satellites. The standard deviations computed with a twice larger thinning interval are almost always larger than those computed with the operational thinning. Such a difference did not appear in the simulated case. This may be due to spatial correlation or maybe inter-channel correlation which are known to lead to underestimating the evaluates.

Conclusions and future directions.

The first iteration of Desroziers and Ivanov's algorithm, cumulating the observations over several dates, has been shown to be able to produce reliable estimates in a simulated case. The estimates seem reasonable in the case of true TEMP messages, its application to ATOVS radiances show several possibly useful and unexpected features but the role of possible correlations has to be clarified.

Future work will extend to the tuning of all observation types and a level by level tuning of B in order to evaluate the impact of this tuning on the analysis and on the forecasts.

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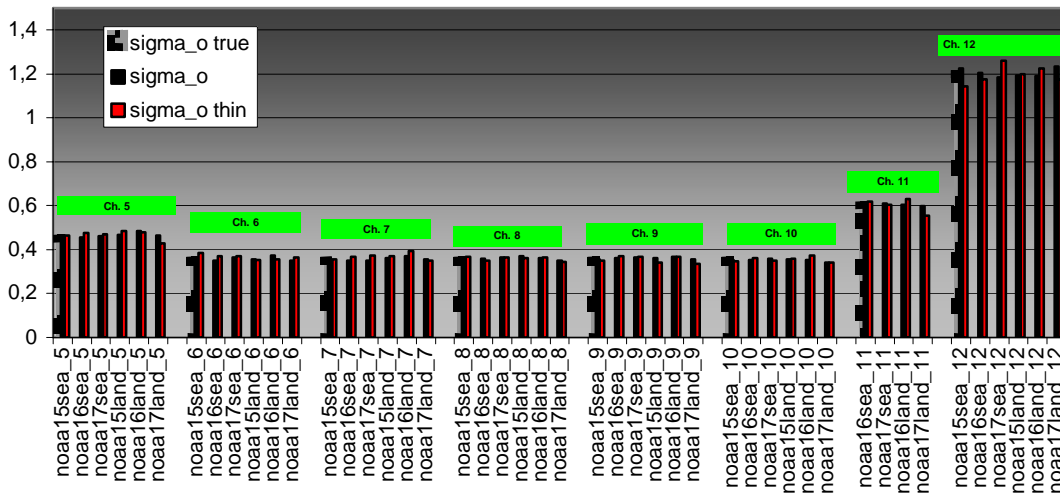In Proceedings of the ECMWF Workshop on Diagnosis of Data Assimilation Systems, 24 November pages 17--28, Reading

Figure. 1: Standard deviations of AMSU A channels obtained by the method in a simulated case. The black bars are computed with the operational thinning between obs. and the red bars with a twice larger thinning interval. A different deviation is computed for each satellite , a difference is also made between sea and land observations. The grey bars with dots show the simulated « true » standard deviations.
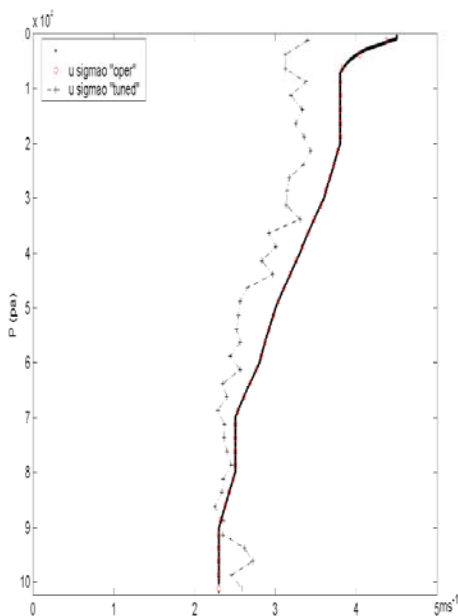

Figure 2: Vertical profiles of TEMP wind speed error standard deviations. The black line with red circle markers is the prescribed profile, the dashed line with "+" markers is the computed profile
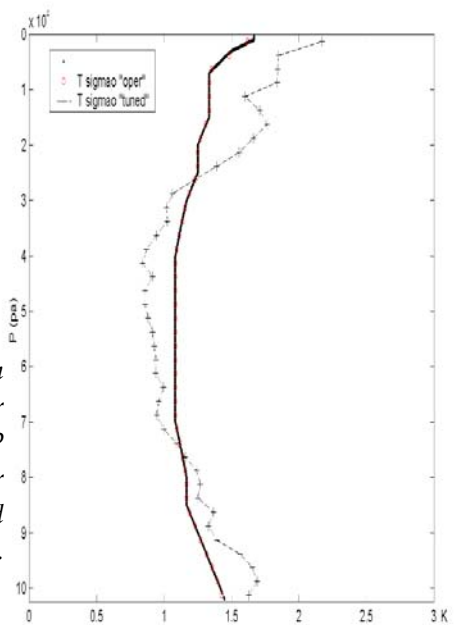
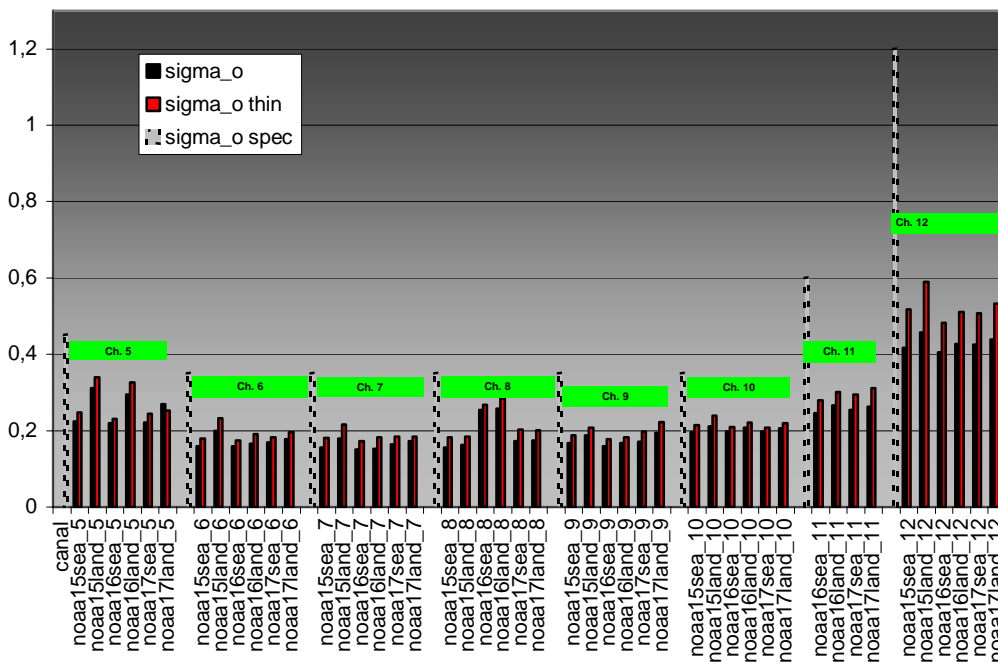*Figure 3: As in Fig. 2 but for TEMP temperature error standard deviation profile.*

Figure 4: Standard deviations of AMSU A channels obtained by the method in a true case. Plotting conventions are the same as in Fig. 1 but this time the grey bars with black dots are the prescribed standard deviations