

Robust Variational Inversion: with simulated ATOVS radiances

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Abstract

Least-square principal is applicable for dealing with data with Gaussian distribution errors, but it is not satisfying with Non-Gaussian distribution errors which are often the feature of the actual observation data. This study examines a "Robust Variational Inversion (RVI)" method based upon the Huber-estimators. On one side, the method is of variational due to the use of the adjoint technique. On the other hand, the method is robust by taking into account the Non-Gaussian feature of data. The method is implemented in the framework of the 1DVAR retrieval with simulated ATOVS data. The preliminary results show the potential of the method.

Methodology

Actually most techniques for retrieval are based upon the least-square principle traced back to Gauss in 1809. The advantage of the least-square method is that the algorithm is easy to be implemented computationally. However, the least-square method is known to be non-robust as the method is very sensitive to the outliers (data with the gross error), and generally needs a strict procedure of quality control to reject the outliers. In fact, some robust methods have been developed in the statistics community since 1960's. For example, a so-called "M-estimators" method from Peter Huber is well known in the "robust statistics" (Huber, 1981).

In remote-sensing retrieval and data assimilation, one generally minimizes a cost function to obtain the best state estimation. For our robust variational inversion algorithm, the cost function can be written as

$$J(\mathbf{x}) = \frac{1}{2}(\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_b) + \frac{1}{2}[H(\mathbf{x}) - \mathbf{y}]^T \mathbf{W}(\mathbf{r}) \mathbf{R}^{-1}[H(\mathbf{x}) - \mathbf{y}] \quad (1)$$

where \mathbf{B} and \mathbf{R} are respectively the background and observation error covariance matrices, \mathbf{x}_b and \mathbf{y} are respectively the background and observation vectors. H represents the observation operator which transforms from x to y space. $\mathbf{W}(\mathbf{r})$ is a diagonal weight matrix with the elements $w(r_i)$ and the normalised departure $r_i = [y_i - H_1(x)]/\sigma_i$ with σ_i the square root of the diagonal elements of the matrix \mathbf{R} . Weight function $w(r) = \psi(r)/r$ with $\psi(r) = dF(r)/dr$ called "influence function" (Hampel, 1986). $F(r)$ is the cost function of observation term. Obviously, for the square cost function $f(r) = r^2/2$, $w(r) = 1$. For non-square cost function that can be derived from Maximum-Likelihood Principal, $w(r)$ will not be 1 any more. Note that the equation (1) is valid only for the case of uncorrelated observation error (i.e., diagonal \mathbf{R} matrix) undertaken in this study.

A descent algorithm generally requires compute the gradient of the cost function at each iteration,

namely

$$\nabla_{\mathbf{x}} J(\mathbf{x}) = \mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_b) + \mathbf{H}^T[\mathbf{W}(\mathbf{r})\mathbf{R}^{-1}(H(\mathbf{x}) - \mathbf{y}) + \frac{1}{2}(H(\mathbf{x}) - \mathbf{y})^T \frac{\mathbf{W}'(\mathbf{r})}{\sigma_o} \mathbf{R}^{-1}(H(\mathbf{x}) - \mathbf{y})] \quad (2)$$

Note that the weight matrix $\mathbf{W}(\mathbf{r})$ should be recomputed with the updated departure \mathbf{r} at each iteration, so the algorithm is also called “Re-Weighted Least-Square” method.

A Robust Variational Inversion Framework

For testing the above algorithm, a robust 1DVAR inversion system originated from the 1DVAR code of ECMWF (Chevallier et al, 2002) is implemented to apply to the ATOVS radiances retrieval problem. A few new developments of the original code were done for this study: (1) the radiative transfer model (RTM) interface is updated from RTTOV6 to RTTOV7; (2) the analysis variables are changed from the original 3 cloud variables profile to the free-cloud temperature T and the specific humidity q profile; (3) adding the robust weight function interface.

The temperature error values are taken from Eyre et al. (1993), and the specific humidity error follows the scheme of Rabier et al. (1998). Fig. 1 shows the background error standard deviation for T and q as well as the error correlation for q . The error correlation is the same for T and q beside that the error correlation of q above 100hPa is removed.

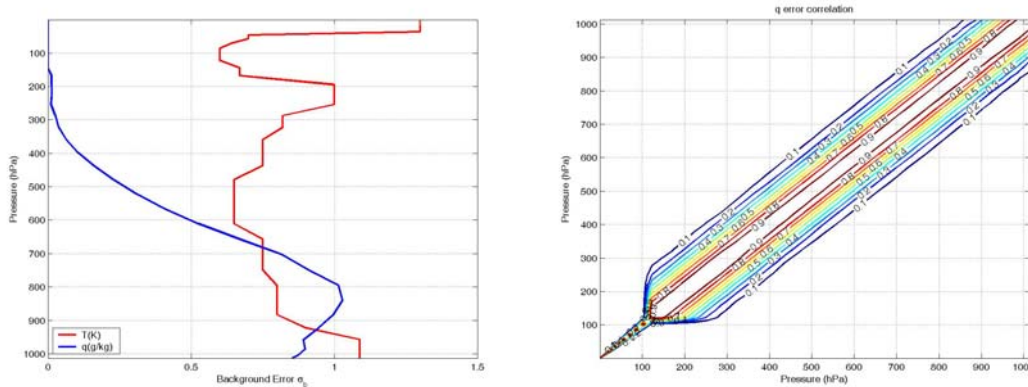


Fig. 1: Specified background error standard deviation for T and q (left panel) and the error correlation for q (right panel).

For observations, we take ATOVS HIRS channels 6, 7, 8, 10, 11, 12 (sensitive to humidity), AMSU-A channels 5-10 (sensitive to temperature) and AMSU-B channels 3-5 (humidity channels). The observation error covariance matrix \mathbf{R} is supposed to be diagonal, and equal to the diagonal elements of the matrix $\mathbf{H}\mathbf{B}\mathbf{H}^T$ that is explicitly computed using the K matrix model of RTTOV. That is, the weight of the background and observations is considered to be equal. The error standard deviation of different channels is shown in Fig. 2. We can see that the error of humidity channels is much larger than that of temperature channels as also shown by Andersson et al. (2000).

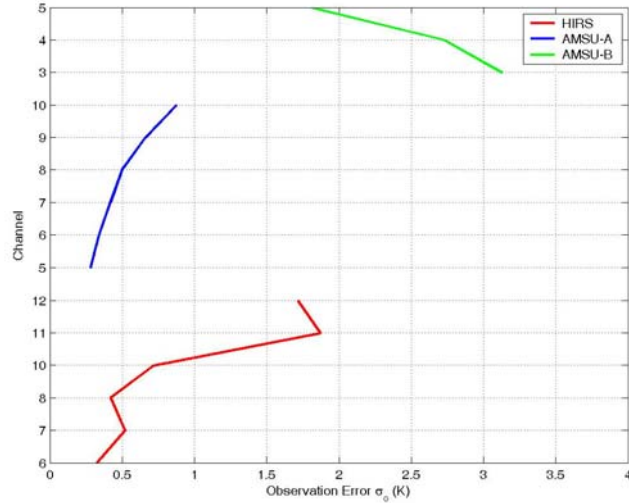


Fig. 2: Specified observation error standard deviation of chosen ATOVS channels which obtained by explicitly computing the matrix \mathbf{HBH}^T .

Some functions are well known to be robust, e.g., Huber, Cauchy, Least-absolute (due to Laplace), Tukey and so on. In this study, Huber's function (Huber, 1964) with the parameter $k=1.345$ is chosen for the robust inversion algorithm both for theoretical and computational reasons. Huber's distribution was theoretically proved to be asymptotically 'least favorable distribution' (Huber, 1964) and the corresponding Huber's function can be considered as the most robust function for various distribution. Computationally, the implementation of Huber-estimator is straightforward and effective. The left panel of Fig. 3 shows the Gaussian pdf and Huber's pdf, Huber's pdf is a long-tail distribution. In practice, Huber's pdf follows Gaussian distribution when $|x| < k$ and follows least-absolute distribution otherwise. The cost function, influence function and weight function related to Huber's pdf are also displayed in Fig. 3 (right panel). For Huber-estimator the weight is equal to 1 in the range of k and $-k$ and decreases with the increase of absolute value of departure beyond this range, while for least-square algorithm the weight is equal to 1 for any departure. We can also see that the influence function is bounded even for large departure. So the robust method reduces the influence of small-probability data on results.

Preliminary results with simulated ATOVS radiances

With the robust 1DVAR Inversion framework, some observing system simulation experiments (OSSE) were run. In these experiments, the "truth" profile is an ECMWF profile taken from the original 1DVAR code. Background states are produced by adding Gaussian random errors (100 realisations), which holds statistically the matrix \mathbf{B} , to the truth profile. Simulated ATOVS bright temperature observations are generated by adding Gaussian or Laplace (long tail) random errors (100

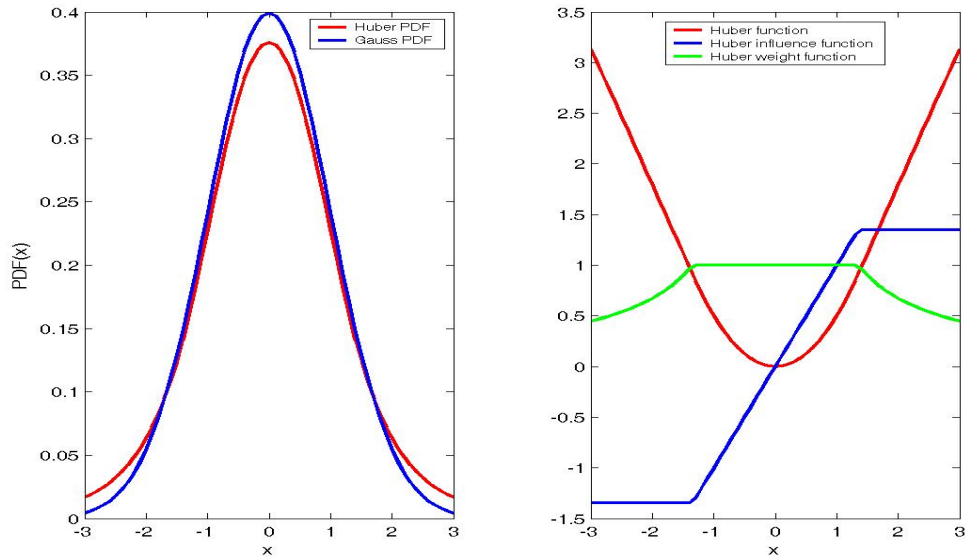


Fig.3: left panel: the Gaussian pdf and Huber's pdf (Huber, 1964); right panel: the cost function, influence function and weight function related to Huber's pdf.

realisations) with the same standard deviation as shown in Fig. 2 to the “truth” ATOVS bright temperatures, which are obtained by applying RTTOV to the “truth” profile.

Firstly, we test the correctness of the normal least-square 1DVAR algorithm upon which our robust 1DVAR algorithm is based. Fig.4 shows the results of the least-square 1DVAR inversion with Gaussian observation errors. Left panel is the root-mean-square (RMS) of the background (red line) and analysis (blue line) error of T (solid line) and q (dash line) for 100 random realizations. One can see a considerable decrease almost at all levels of RMS of the analysis error relative to the background error both for T and q. Correspondingly, the decrease of the analysis error is also observed in bright temperature space as shown in the right panel of Fig.4.

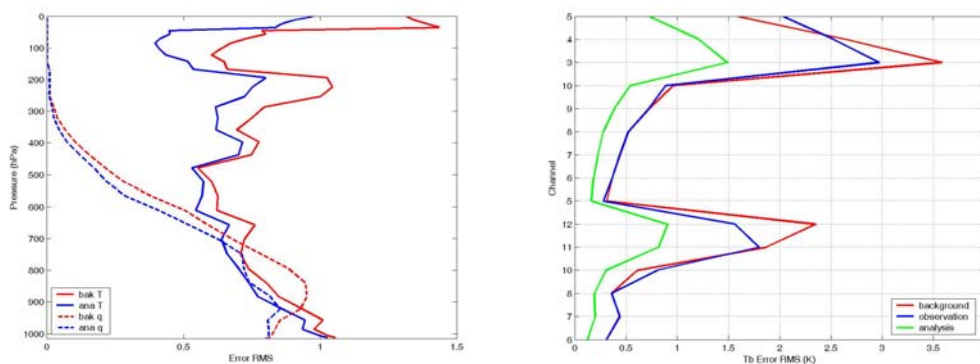


Fig. 4: left panel: the averaged background and analysis error RMS of 100 realizations with Gaussian observation error and the least-square method. right panel: the background, observation and analysis error RMS in observation space.

Secondly, a comparison between the robust and least-square algorithm with Gaussian observation errors is performed. We know from the Maximum-Likelihood Principle that the least-square method should be the best one for Gaussian distribution error. In generally, the robust method is required close to the results of the least-square one even for the case with Gaussian error. Fig. 5 gives the results of comparison of the analysis error RMS for the robust (red line) and least-square (blue line) algorithms with the Gaussian observation error both in (T, q) space and bright space. As expected, the robust result is very close to the least-square one. It seems that humidity at low levels, where the decrease of humidity retrieval precision for the robust algorithm is larger, is more sensitive to algorithm.

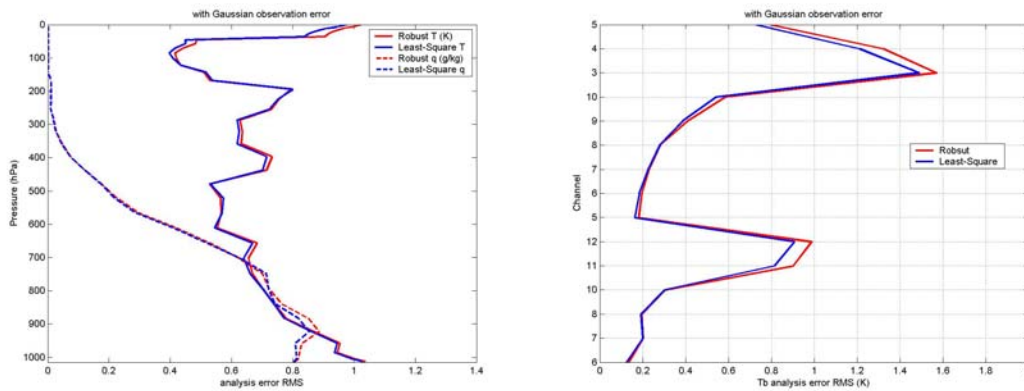


Fig. 5: Comparison of the analysis error RMS for the robust and least-square algorithms with the Gaussian observation error, respectively in analysis-variable space (left panel) and observation space (right panel).

Finally, we compare two algorithms for the case with Laplace observation errors (long tail distribution) with the same standard deviation as Gaussian one. The results are shown in Fig. 6. One can see that the robust result is slightly better than the least-square one, particularly for specific humidity q and for the humidity channels in observation space. For temperature retrieval, the robust method does not show the advantage for this case. This is not well understood and need to be further studied. Possible reasons contain tuning of algorithm parameters such as k parameter of Huber's function, feature of simulated error observations and convergence rate of algorithm and so on.

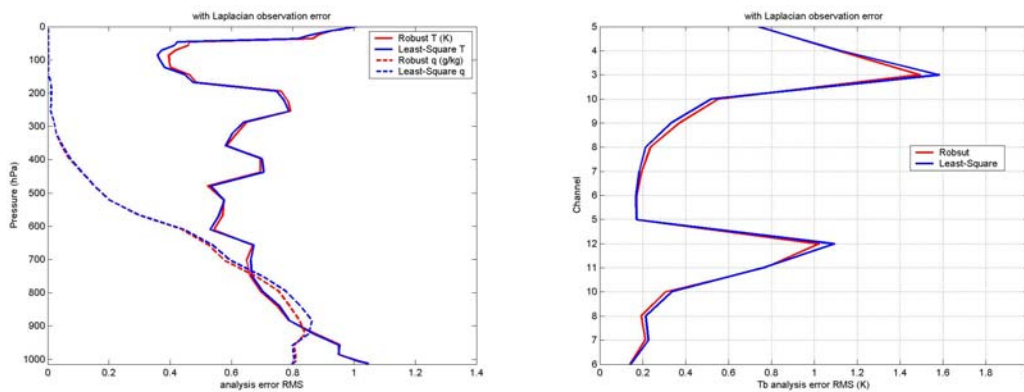


Fig 6: Same as Fig 5, but with Laplace observation errors (long tail distribution).

Summary

A general robust inversion method is formalized and a robust 1DVAR framework is implemented for retrieval of (T, q) profile from ATOVS radiances. Preliminary test of robust algorithm with Huber weight function indicates the potential of the method. Further study in detail is needed to compare with other weight function and to apply to real data.

Acknowledgement

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