

Resolving Tropical Storm Inner Core Temperatures with a Three-Meter Geostationary Microwave Sounder

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abstract

Geostationary (GEO) weather satellites have differed from low-Earth orbiting (LEO) weather satellites in that they have not carried microwave radiometers. The reason is that to get LEO resolution from GEO, one needs an antenna forty times as large. Even scaling the small AMSU antenna requires a 6-meter dish. As discussed elsewhere, a 3-meter antenna still provides adequate spatial resolution for measuring precipitation and an initial guess for infrared soundings in cloudy conditions.¹ The higher temporal sampling possible from GEO also permits improved monitoring of severe storms, but an objection to developing a GEO microwave sounder has been that tropical storm inner core temperature profiles would not be adequately resolved with such an antenna.

This paper demonstrates how inner core temperatures may be measured from GEO using a 3-meter antenna and post-processing. Because this trades sensitivity for resolution, radiometer system noise must be as low as possible. Other sources of error sources imperfect boundary temperature knowledge and antenna pattern uncertainty. Simulations and covariance analyses suggest that the required system noise temperatures, although not yet commercially available, are not beyond the state-of-the-art. Although one might have to choose between inner core temperature soundings and hourly full-disk soundings, it would eliminate one more objection to flying a modest size microwave radiometer at geostationary altitude.

Problem

A microwave sounder has been proposed as a Pre-Planned Product Improvement (P³I) for the GOES-R series of geostationary environmental satellites. The first GOES-R satellite is to be ready for launch in 2012, but the microwave sounder would probably not be ready until 2015. It is to provide temperature and humidity soundings, precipitation, sea surface winds and tropical storm inner core temperature profile. Its performance requirements are summarized in Table 1. While the temperature, humidity and precipitation requirements are relatively easy to meet, the sea surface wind and inner core temperature requirements are more difficult. Possible solutions to the inner core problem are discussed here.

The term “warm core anomaly” refers to the increased temperature around the eye of a tropical storm. At the 250 mb pressure level, the temperature can be as much as 20 K higher than its surroundings. This temperature elevation indicates storm strength and can only be measured

remotely using microwave radiometers. Although soundings are typically done at 60 GHz to get surface temperatures, inner core temperatures are also observable at 118 GHz because of their altitude.²

Table 1. GOES-R Microwave Sounder Requirements

	Temperature Sounding	Humidity Sounding	Precipitation	Sea Surface Winds	Warm Core Profile
Resolution (km)	100	30	30	25	20
Sensitivity (K)	1	1	1	1	1
Frequency (GHz)	54-60	183	118-183	20	54-60
Bandwidth ³ (MHz)	200	500	500	200	200

Even with large antennas, resolving inner core profiles is difficult. This can be seen from simulations where horizontal temperature distribution is modeled using a Gaussian function whose peak value decreases by 1/e at a radial distance of 20 km

$$T(r) = T_{\infty} + \Delta T \cdot \left(1 - e^{-r_c^2/r^2}\right) \quad (1)$$

This formula is based on a paper by Greg Holland.⁴ Here T_{∞} is the background temperature, ΔT is the warm core temperature anomaly, r_c is the core radius, and r is the radial distance from the core center (x_0, y_0)

$$r^2 = (x - x_0)^2 + (y - y_0)^2 \quad (2)$$

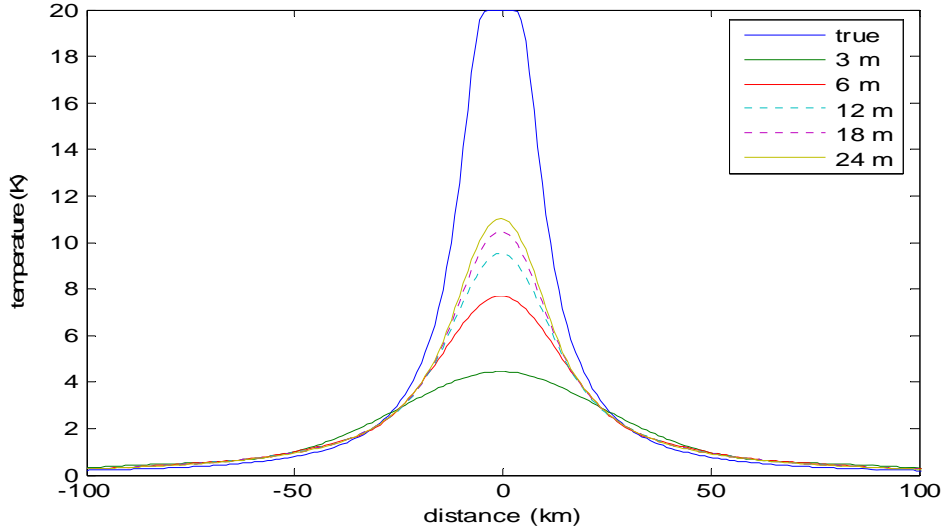
Figure 1 shows that a 6-meter antenna operating at 60 GHz from GEO captures less than 40% of the warm core temperature anomaly from GEO altitude. Even a 24-meter antenna recovers only 55% of the peak temperature. Launch vehicle size and the need to control antenna dimensions to a fraction of a wavelength, however, make 3 meters about the largest practical diameter for an antenna operating at up to 183 GHz.

Hypothesis

If we had a 3-meter antenna at GEO and could oversample the inner core region, we might be able to estimate its temperature profile. On the surface, that might appear to violate the diffraction limit, but diffraction applies to a fixed view and does not consider signal strength⁵. Because of numerical error, post-processing trades signal strength for spatial resolution. Rather than give up on a GEO

microwave sounder due to the difficulty of flying a large, tightly-controlled antenna, we would try to improve sensitivity and then trade surplus sensitivity for spatial resolution.

Figure 1. Warm Core Temperature Profiles Resolved from GEO



Sensitivity σ_T depends on system temperature T_s , integration time τ and bandwidth B

$$\sigma_T = T_s / \sqrt{B\tau} \quad (3)$$

One way to improve sensitivity is to reduce system noise temperature. This can be done by amplifying the signal with a low-noise amplifier as soon as it is received. A second is to increase the integration time, *i.e.* scan more slowly. This not only reduces the noise but increases the signal. A third is to increase bandwidth. The greater the bandwidth, the more random the noise and the quicker it integrates to zero. If the system temperature were 1000 K, the integration time were 1 second, and the bandwidth were 100 MHz*, scan time would be under a minute and the sensitivity would be 0.1 K. If the antenna field-of-view and inner core width were 100 km and 20 km respectively, it would take 6 twenty-kilometer samples to scan across the inner core. In two dimensions, that would be 36 samples. At one second a sample, measuring the inner core temperature profile could take less than a minute.

In a sense, this is an estimation problem. Given noisy observations of the warm core anomaly, we are asked to determine its amplitude. Depending on what is known from visible and other observations, we would solve for one or many states. Post-processing options could include:

1. scaling the observed temperature peak
2. solving for inner core temperature model parameters
3. deconvolving to recover the actual temperature profile⁶

* The narrow bandwidth AMSU stratospheric sounding channels are not needed in a geostationary microwave instrument.

All three approaches assume that the antenna pattern is known. In addition, the first assumes that the shape of the temperature profile is known, *i.e.* it only changes in amplitude. The second assumes that the spatial temperature distribution can be parameterized and one estimates a few parameters which include the temperature peak. The third estimates the full spatial distribution of brightness temperature throughout the scene. Because of the broad spatial extent of the antenna pattern, there are always more object points that contribute to the image than there are observations. Solving for the object points requires *a priori* information, such as might come from a preliminary scan.

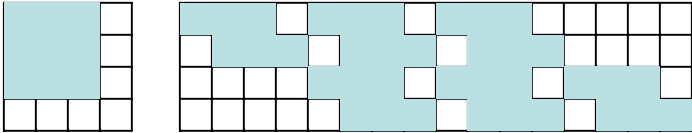
Design

An essential first step in the simulations is to produce a brightness temperature image Y from the brightness temperature object X . Although we could use commercial convolution routines, we need a record of everything that has been done for the deconvolution method. So, we develop our own convolution method. If the columns of the image and object are concatenated to produce vectors \vec{y} and \vec{x} , the convolution becomes a matrix multiplication

$$\vec{y} = G \cdot \vec{x} \tag{4}$$

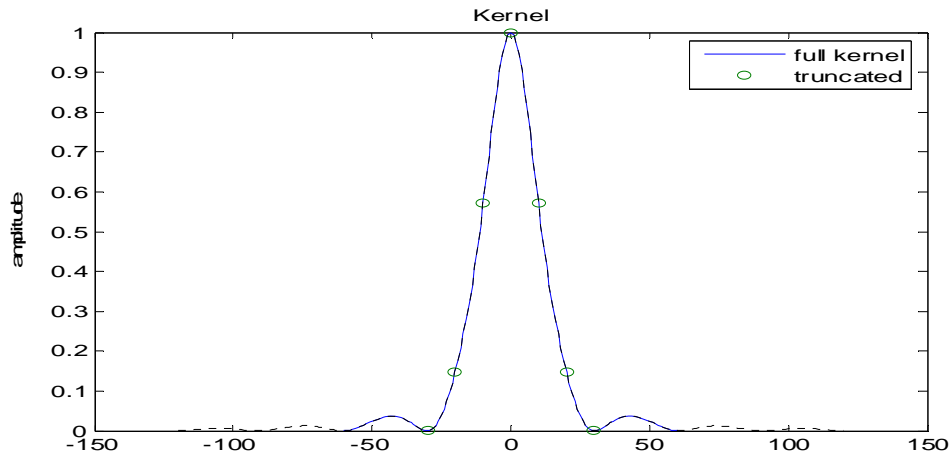
The convolution matrix G represents the antenna pattern or sampling kernel. The size of the convolution matrix is M^2 -by- N^2 where M and N are the size of the image and object respectively. The kernel, object and image matrices are all square. As an example, Figure 2 shows a 3-by-3 kernel superimposed on a 4-by-4 object. The valid image is only 2-by-2 because there are only two vertical and two horizontal positions where the kernel does not extend beyond the ends of object. The 4-by-16 convolution matrix G is shown on the right. The rows of G are the columns of the sampling kernel, and each successive row shifts one to the right until the kernel reaches the bottom of the object. Then, the kernel jumps to the top of the object matrix and shifts one column to the right. The convolution matrix is band diagonal.

Figure 2. Object and Convolution Matrix



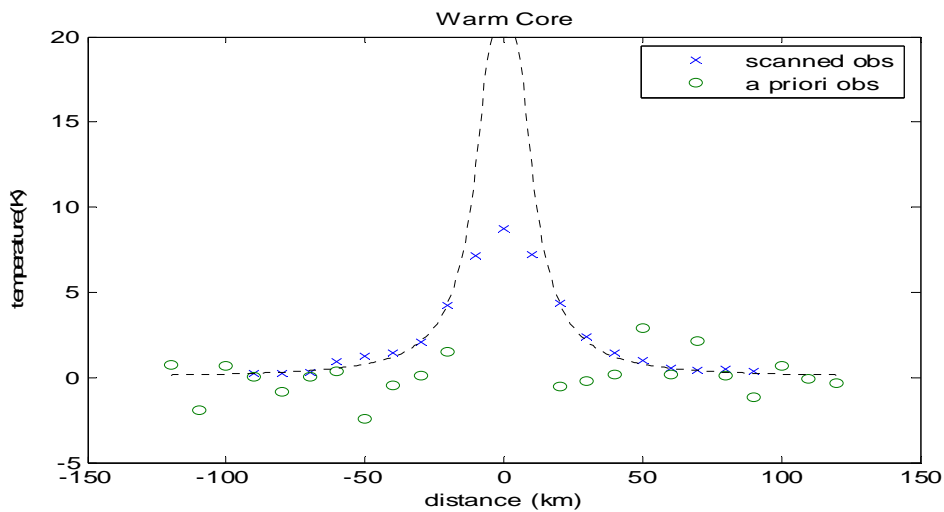
Convolving the kernel with the object gives an image. Although the kernel, object and image all have one kilometer resolution, this is unnecessary for our processing. So, we pixelate to a ground sample distance of our choosing. Figure 3 shows the antenna pattern used to generate the truth data in solid blue. It goes out to the second nulls. The pixelated pattern used for modeling was sampled at the green circles and taken only to the first null. This was done to be conservative and to reflect some of the inevitable uncertainty as to the actual antenna pattern.

Figure 3. True and Assumed Antenna Patterns



Recall that the a priori observations are included to provide temperature information around the edges of the scanned region. They are not needed near the inner core and are inaccurate due to the sharp peak there. For this reason, we mask out those a priori observations as shown in Figure 4. The blue x's are the scanned observations, and the green o's are the *a priori* observations.

Figure 4. Scanned and A Priori Observations



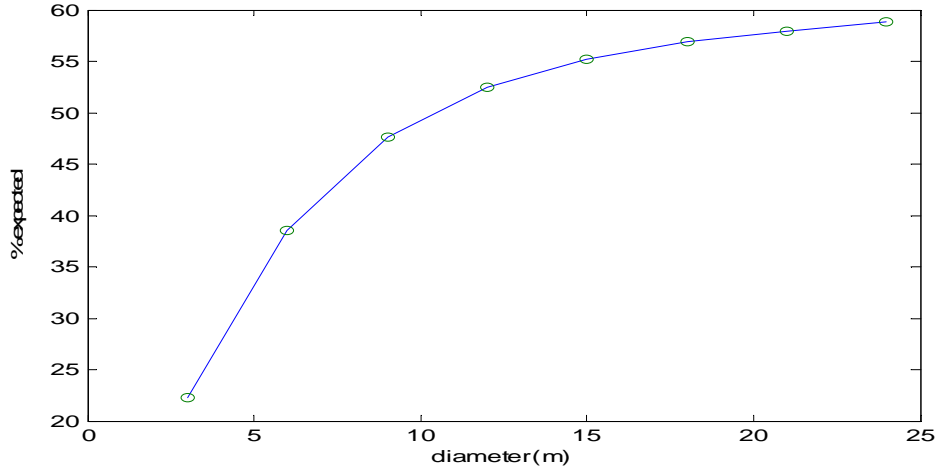
In the first method, we assume that the inner core temperature profile follows the Holland model and only estimates its amplitude. This assumes that the background, location and radius are known, perhaps from visible observations. If the inner core location and size are approximately known, the background can be computed from preliminary scan results by leaving out the inner core region. Visible observations are typically geolocated to kilometer accuracy. So, if the warm core anomaly occurs at the center of the eye, it should be possible to locate it from visible observations to kilometer accuracy as well. Inner core radius is apt to be more difficult because the core size seen with visible light may not be the same as that seen with microwaves. If the radius is known, the

observed temperature elevation ΔT_o can be scaled according to the fraction of the warm core anomaly f it is expected to represent

$$\Delta T = \Delta T_o / f \quad (5)$$

Expected fractions for a ten kilometer radius are shown in Figure 5.

Figure 5. Expected Fractions



In the second method, we still assume that the brightness temperature profile follows the Holland model. Now, though, we estimate the inner core location, radius and temperature elevation. To choose between candidate profiles, we compute a loss function J equal to the sum of all the squared brightness temperature differences between the observed image temperature vector \vec{y} and the image temperature vector \vec{g} computed from the candidate profile

$$J = (\vec{y} - \vec{g})^T (\vec{y} - \vec{g}) \quad (6)$$

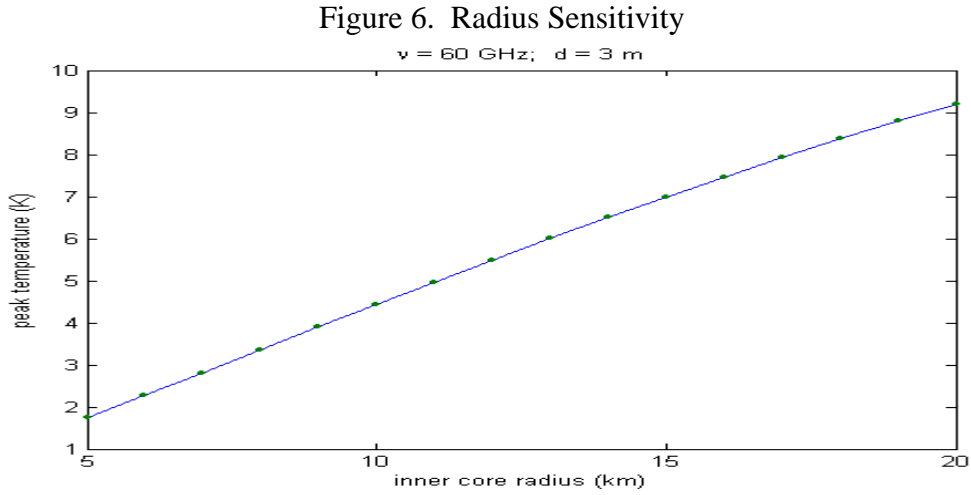
The profile that gives the smaller loss function is chosen over the one that gives the larger one.

For the third method, we again minimize the sum of the squared observation errors, but this time we have less accurate a priori observations as well. We use the Newton-Raphson method with the convolution matrix acting as the derivative. The brightness temperature state vector \vec{x} , given an a priori estimate \vec{x}_0 having covariance S_0 and observations \vec{y} having covariance W is⁷

$$\vec{x} = \vec{x}_0 + (S_0 + G^T W G)^{-1} G^T W (\vec{y} - \vec{g}) \quad (7)$$

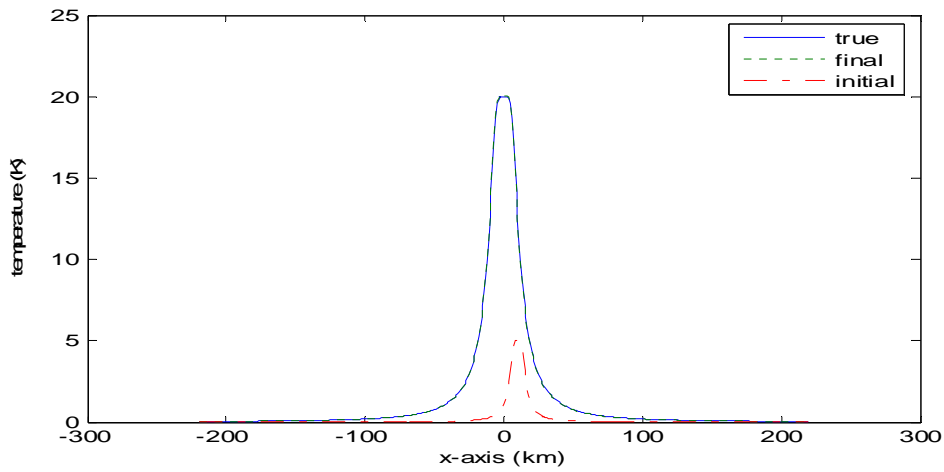
Observations

The first method, *i.e.* scaling, works as long as the core location and radius are known. If the assumed radius is high or low, we under- or overestimate the warm core temperature anomaly. Figure 6 shows the effect of error in the assumed inner core radius for a 3-meter antenna operating at 60 GHz. Over the range of radii from 5 to 20 kilometers, the observed peak increases at a rate of 0.5 K per kilometer. For a 10 kilometer core radius, the scaling factor is about 4, and a one kilometer error in the assumed core radius gives a 2 K error in the temperature elevation. The suitability of this method depends on the accuracy with which inner core radius can be known by other means.



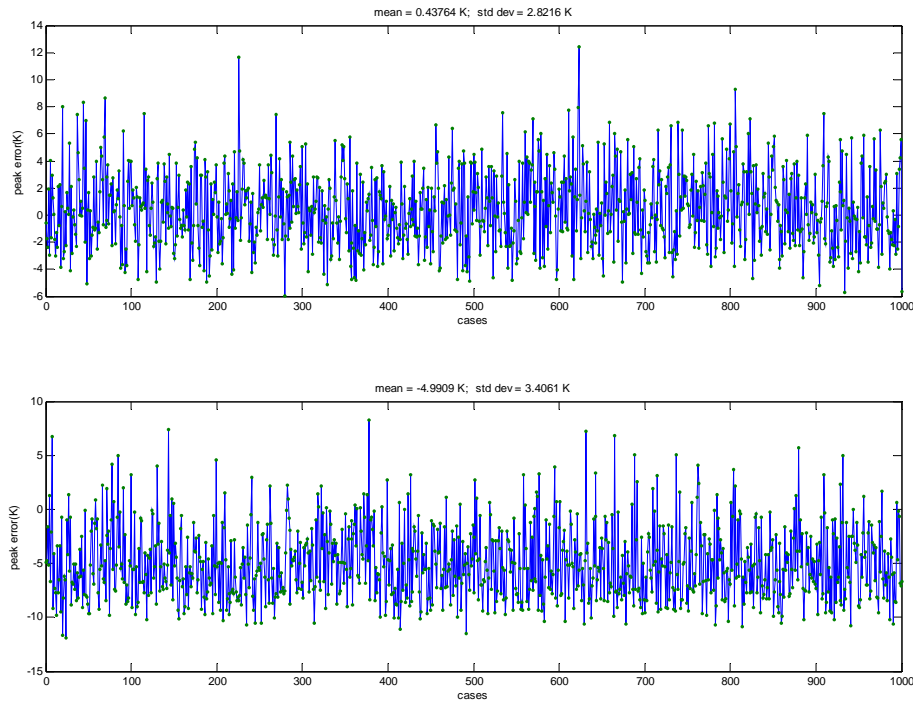
The second method, *i.e.* four-parameter estimation, does quite well. It converges from a poor initial guess as shown in Figure 7. It also matches the temperature elevation to better than 1 K accuracy. It is more reliable than scaling because it does not depend on precise knowledge of the core location and radius from visible measurements. This method does depend on the inner core temperature profile following the Holland model. How robust it is to such errors needs further study.

Figure 7. Method 2: Four-Parameter Estimation



The third method makes no assumptions about the inner core structure. Figure 6 shows the peak recovery error for one thousand cases. The bottom plot is for 60 GHz and 20 km ground samples. The mean error is 5 K and the standard deviation is 3.4 K. Finer sampling does not help. The top plot is for 118 GHz and 10 km ground samples. The mean error is 0.4 K and the standard deviation is 2.8 K. However, if the 118 GHz oxygen absorption band could be used, the inner core measurements obtained with a 3-meter antenna should be quite satisfactory. Using 118 GHz is conceivable because the inner core temperature elevation occurs at 250 mb (10 km). Above that altitude, ice crystal scattering is not the problem that it is for low altitude sounding.

Figure 6. Deconvolution Error at 118 GHz and 60 GHz



Conclusions

Even though the 60 GHz beamwidth for a 3-meter antenna at GEO is much wider than the inner core of a tropical storm, the warm core temperature elevation can be estimated. Three methods were considered here. All three require knowledge of the antenna pattern and oversampling. The first method estimates the warm core anomaly by scaling the observed profile and works if the inner core radius is known. The second estimates the four model parameters, *i.e.* temperature anomaly, inner core radius and location, and is accurate to 1 K as long as the form of the inner core temperature profile matches the model. The third makes no assumptions about structure, estimates the entire temperature profile and captures the 20 K temperature anomaly assumed here with a mean error of 5 K. If 118 GHz can be used instead of 60 GHz, the mean error falls below 0.5 K. Noise at any given point either at 60 and 118 GHz is about 3 K. This initial investigation suggests that the warm core temperature anomaly of tropical storms can be measured from geostationary orbit with an antenna of moderate size.

Acknowledgements

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