Vertical Covariance Localization for Satellite Radiances in Ensemble Kalman Filters

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What is an EnKF?

- For both the variational and Kalman filter approaches to data assimilation, the analysis equation looks like

$$\mathbf{K} \equiv \mathbf{P_b H^T} \left(\mathbf{H P_b H^T} + \mathbf{R} \right)^{-1}$$

(N x p) (p x p)

$$\vec{x}_a = \vec{x}_b + \mathbf{K}\left(\vec{y} - \mathcal{H}(\vec{x}_b) \right)$$

- In (unlocalized) EnKFs, the background error covariance is replaced by the sample error covariance below, where the k columns of X are the perturbation state vectors of each ensemble member

$$\mathbf{P}_j^f \equiv \mathbf{X}_j \mathbf{X}_j^T$$

(Nxk) (kxN)

Covariance Localization

- **Spurious correlations limit the utility of unlocalized EnKFs**, especially when the number of ensemble members is much less than the number of state variables. The sample error covariance can be localized
- in **model space (Eq. 1)**

$$\mathbf{K}_j = \left[\left(\rho \circ \mathbf{P}_j^f\right)\mathbf{H}^T\right]\left[\mathbf{H}\left(\rho \circ \mathbf{P}_j^f\right)\mathbf{H}^T + \mathbf{R}\right]^{-1}$$

- or in **observation space (Eq. 2)**

$$\mathbf{K}_j = \left[\rho \circ \left(\mathbf{P}_j^f \mathbf{H}^T\right)\right]\left[\rho \circ \left(\mathbf{H}\mathbf{P}_j^f \mathbf{H}^T\right) + \mathbf{R}\right]^{-1}$$

Radiance Localization Difficulties in the Vertical

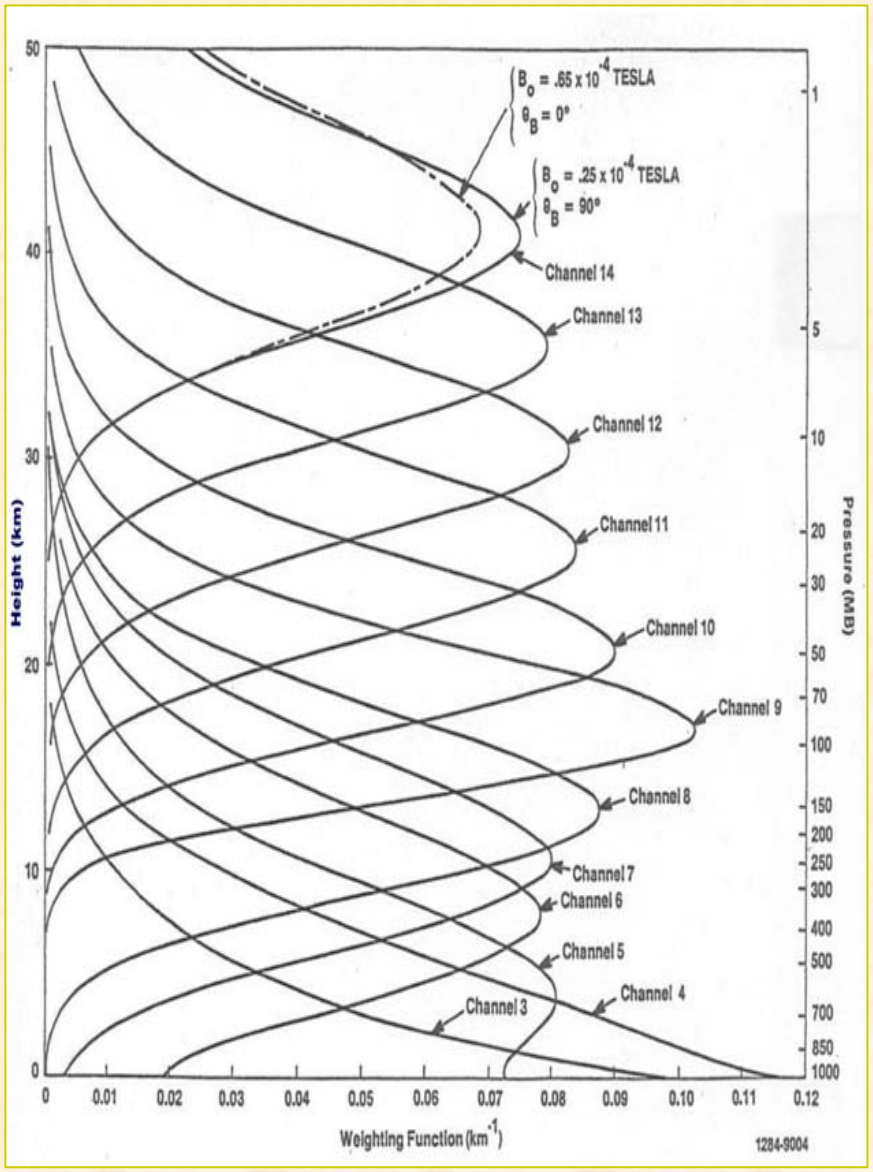
We will show why a widely-used covariance localization method for EnKFs has difficulty with satellite radiances:

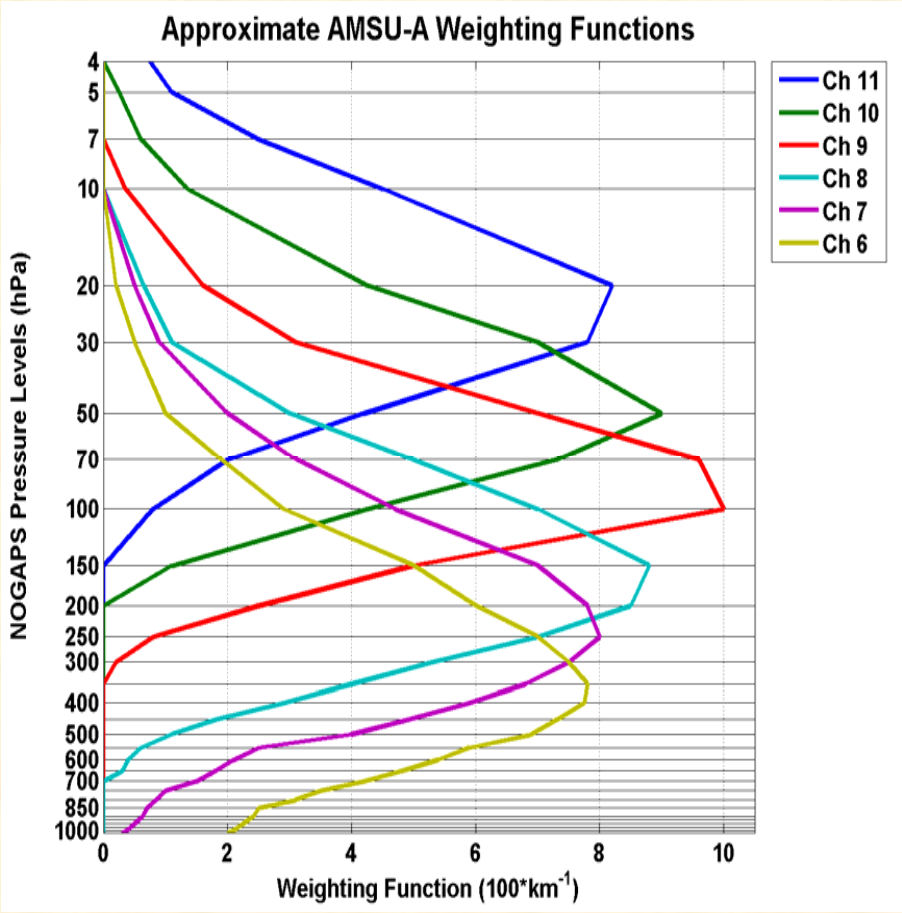
1) **Integrated measures have no well-defined location.**
2) **Because neighboring satellite channels have broad, overlapping weighting functions, the radiance perturbations measured by these channels are correlated; any localization that removes these correct correlations will be suboptimal.**
3) **If the localization in the vertical is significantly broader than the radiance correlation length, spurious correlations will remain; if it is *not* significantly broader, *correct* correlations will be eliminated. Because typical weighting functions cover a significant fraction of the model atmosphere, *there may be no middle ground*.**

1D Model

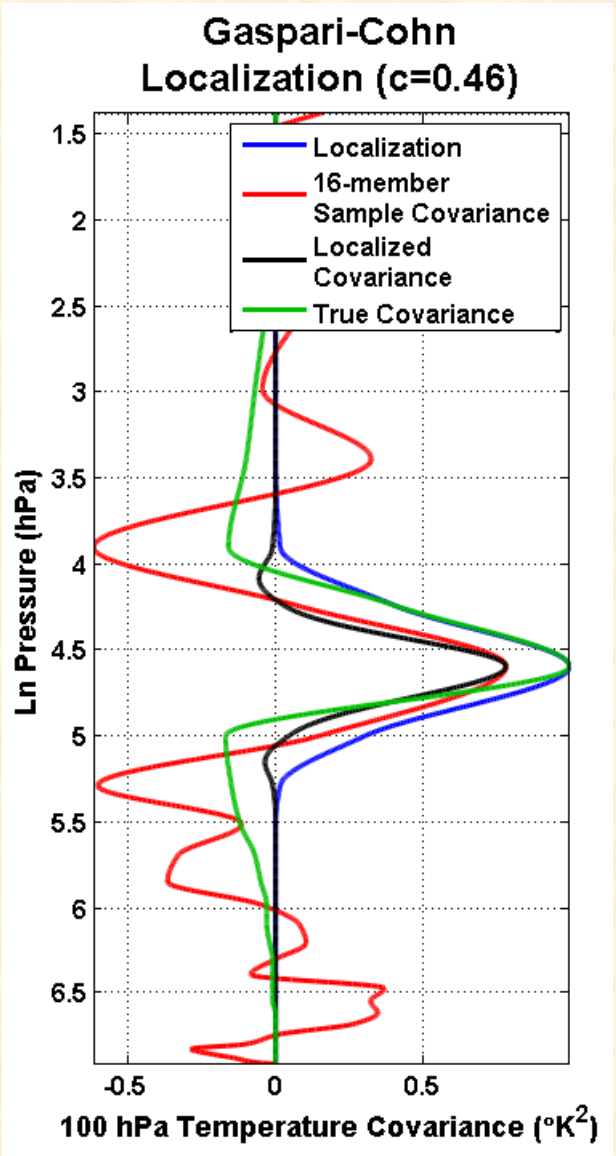
- 30 NOGAPS levels
- NAVDAS forecast error covariance for temperature (assume variance of 1.0 K^2)
- AMSU-A analogs to channels 6-11
- Localization widths are tuned to give the best results for each observation error variance, ensemble size, and type of localization

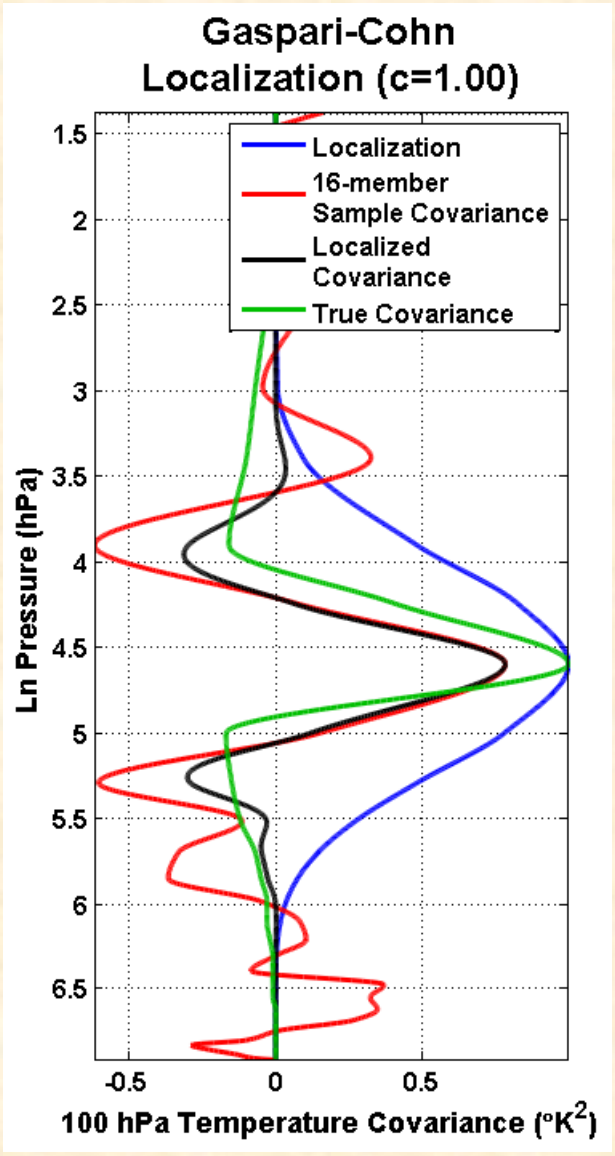
AMSUA-A Analogs

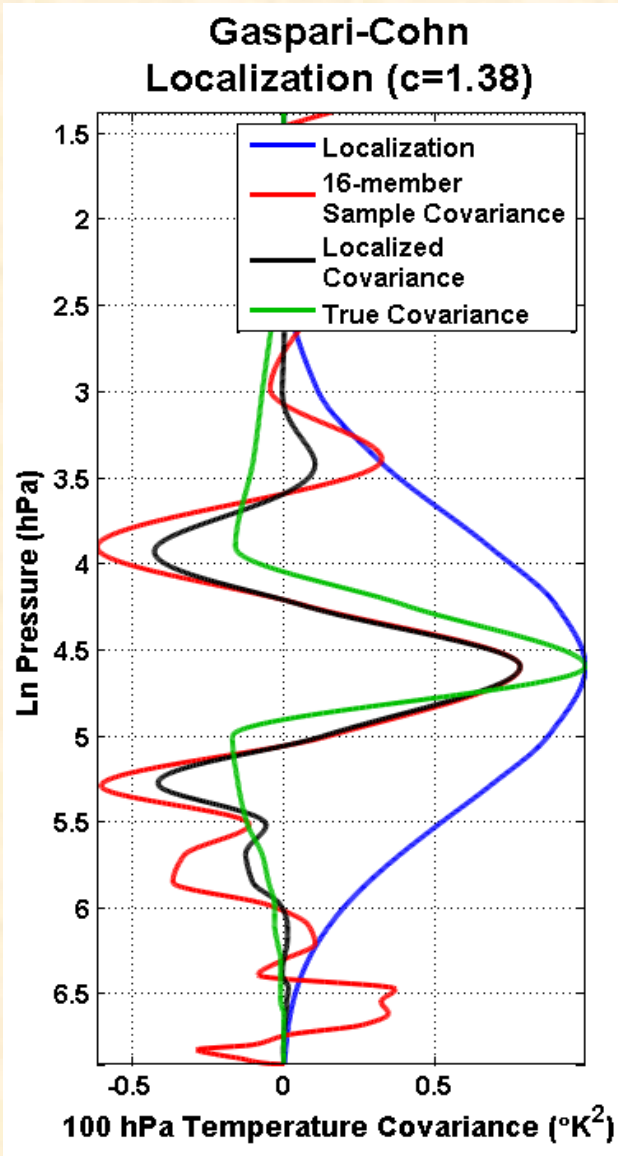




Temperature Localization



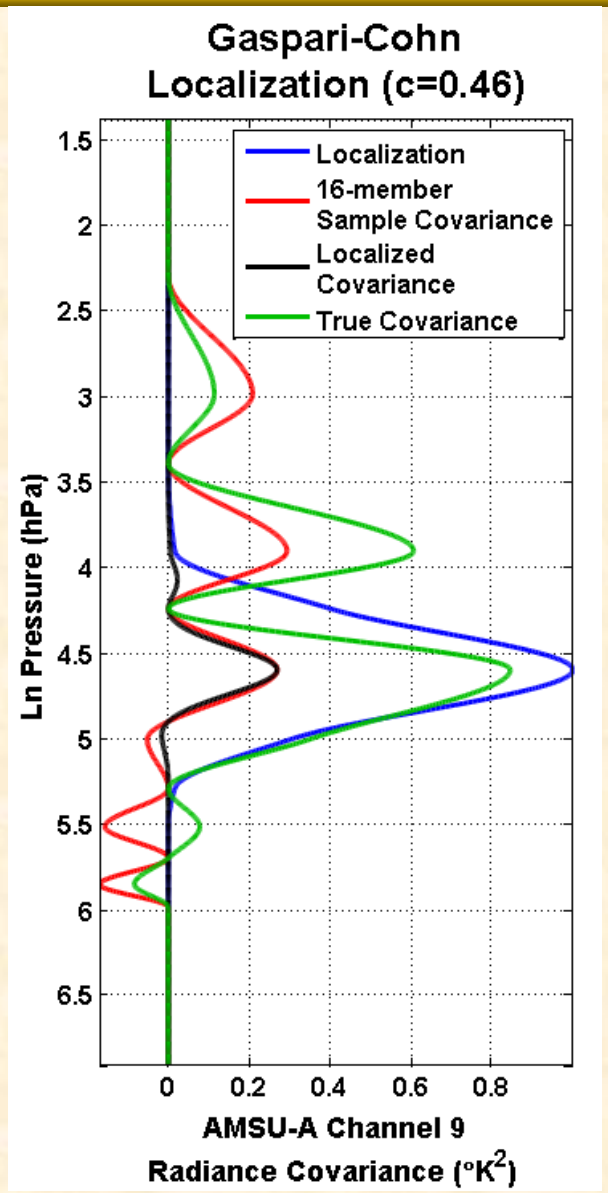


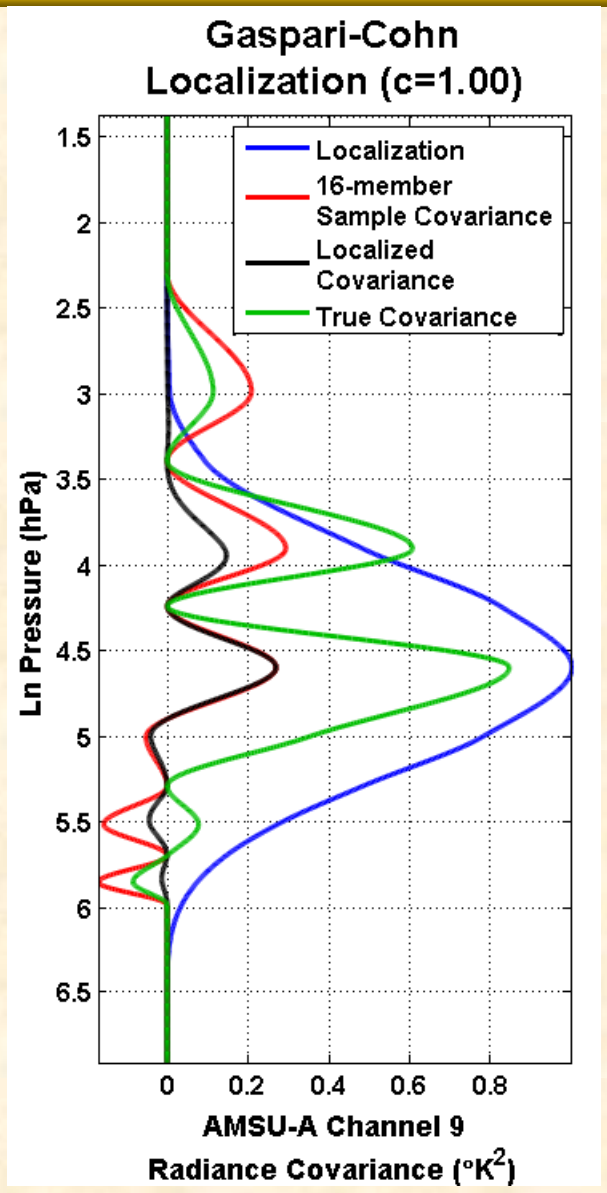


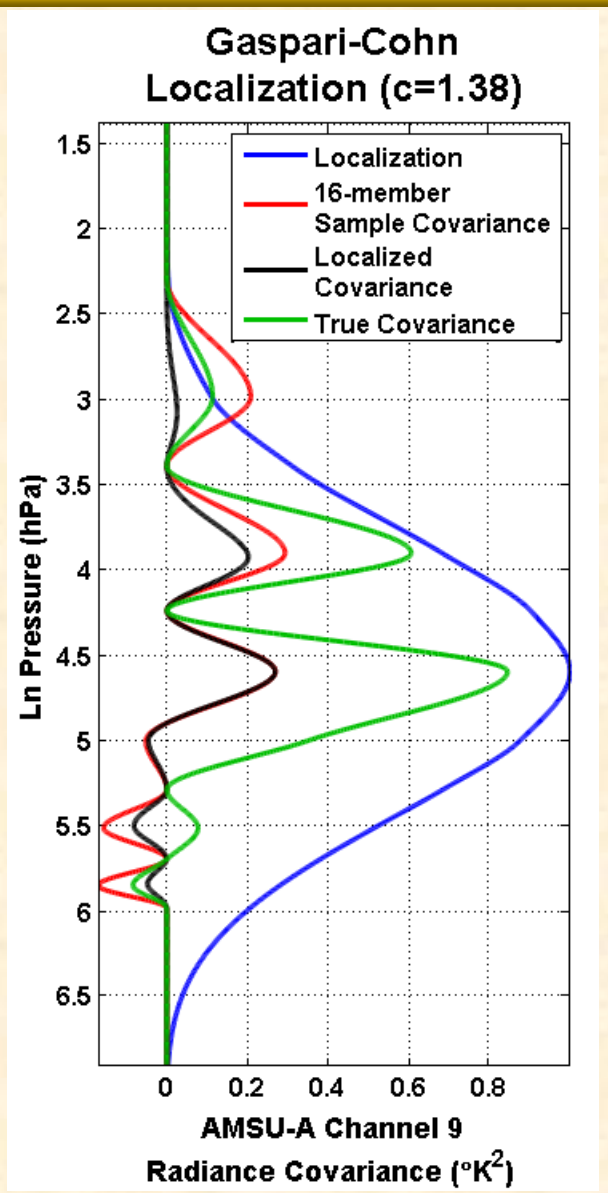
Distance in Radiance Space

- In model space, distance is well-defined
- What is the distance between channel 1 and channel 2?
- One commonly used approximation is the distance between the peaks of the weighting functions—the radiances in each channel are assigned to a particular location in model space
- The result is that after observation space localization, each channel can correct only one model level

AMSU-A Channel 9 Radiance Localization

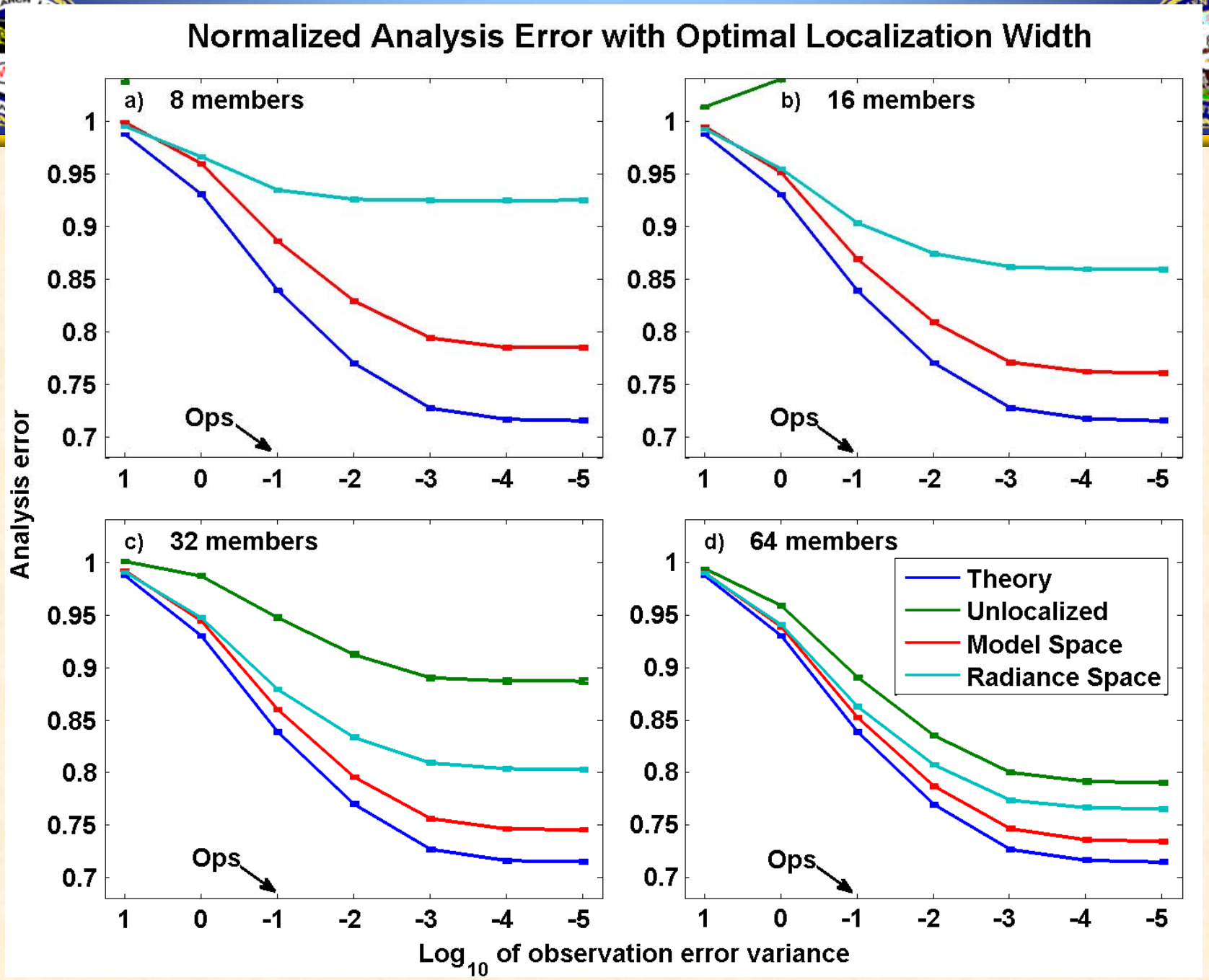






Experimental Design

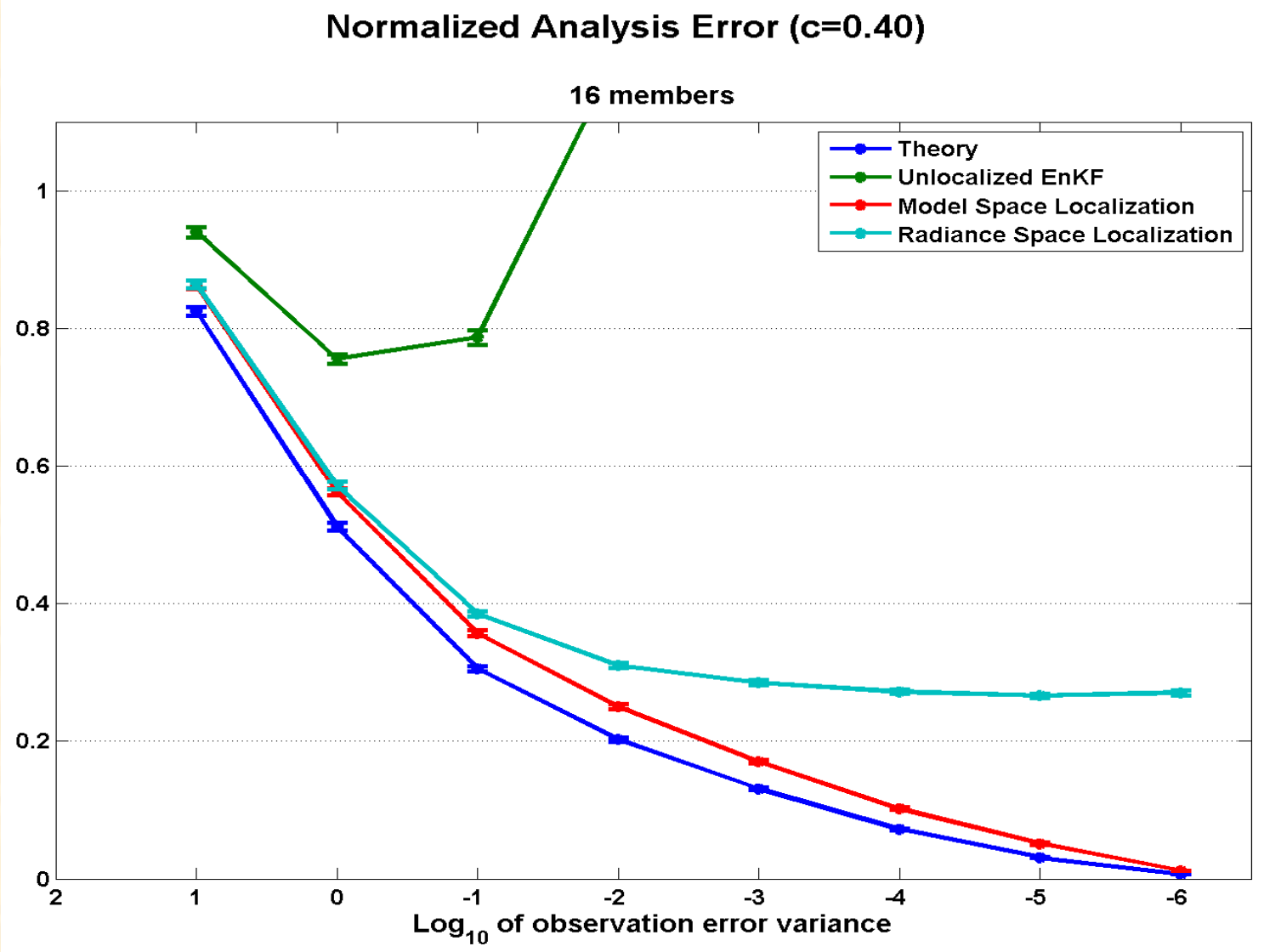
- 100,000 trials, 4 ensemble sizes (8, 16, 32, and 64), 7 observation error variances (10^{1}, 10^{0}, 10^{-1},... 10^{-5})
- For each trial, a forecast error and observation error are generated, and the localization methods are applied with a range of localization widths (from 0.1 to 2.3 units in Log_{10} pressure (hPa))
- The mean square analysis error normalized by the mean square forecast error, averaged over all trials, is plotted against the log of the observation error variance for each ensemble size for the optimal localization width
- 99% confidence intervals are shown


Normalized Analysis Error with Optimal Localization Width
a) 8 members
b) 16 members
c) 32 members
d) 64 members
Ops
Theory
Unlocalized
Model Space
Radiance Space
Analysis error
Log10 of observation error variance
1
0.95
0.9
0.85
0.8
0.75
0.7
1
0
-1
-2
-3
-4
-5

Idealized Instrument

- Given as many independent satellite radiances as vertical levels, the analysis error should tend to zero as the observation error variance tends to zero
- 30 AMSU-A analogs with overlapping weighting functions
- Look at the 16-member case, which has insufficient rank for the unlocalized EnKF to be useful, and repeat the experiments

Normalized Analysis Error



Summary

- Distance and location are not well-defined for integrated measures
- Broad satellite weighting functions force observation space localization functions to either be so broad that they lose effectiveness, or so narrow that true inter-channel covariances are suppressed or eliminated
- Both problems contribute to the systematically worse performance of observation space localization in our 1D experiments
- Failure to convergence to zero analysis error in the presence of perfect observations is another troubling feature of observation space localization

Conclusions

- Although observation space localization is significantly more computationally efficient than model space localization, there are existing methods that are computationally feasible and localize in model space (e.g. Buehner 2005, Bishop and Hodyss 2009)
- We recommend that users weigh carefully the computational performance gains they expect relative to the drawbacks demonstrated here

- Campbell, W. F., C. H. Bishop, and D. Hodyss, 2010: Vertical covariance localization for satellite radiances in ensemble Kalman filters. *Mon. Wea. Rev.*, **138**, 282-290.

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