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Information content of radiance climatologies

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Acknowledgements

- Sean Healy, ECMWF
- Andrew Collard, NCEP
- Fiona Hilton, Met Office

- Tim Hultberg, EUMETSAT



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Outline

- Motivation
- Optimal estimation of radiances
- Consequences for retrieval accuracy
- Conclusions



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Motivation (1)

- Assimilation of radiances in NWP:
 - normally 2 sources of information:
 - measured radiances
 - NWP background
 - Now we have 3rd source - “radiance climatology”:
 - a large ensemble of “historical” radiances from the same instrument, used to compute the PCs
- Question: What are the implications of this, for information content, analysis error, etc.?



Motivation (2)

- The “engineering” properties of the PCs of spectra from advanced IR sounders have been demonstrated:
 - More compact representation of information in full spectrum
 - Faster computations for forward and inverse problems
 - Noise reduction – cleans up the spectrum – potentially important when carefully selected small spectral windows must be used
- ... but do the “radiance climatologies” used to compute these PCs also contain information to improve the retrieval, when all/most channels are used?



Optimal estimation of radiances (1)

radiance information

observation:	\mathbf{y}^o	error cov, \mathbf{N}
radiance climatology:	mean, \mathbf{y}^m	covariance, \mathbf{C}

definitions

$\mathbf{C}_o = \mathbf{C} + \mathbf{N}$ = climatological covariance including noise

$\mathbf{N}^{-1/2} \mathbf{C}_o \mathbf{N}^{-1/2} = \mathbf{E} \mathbf{\Lambda} \mathbf{E}^T$; eigenvector rep. in noise-normalised space

Change space: – noise-normalise, rotate, truncate

observation:	$\mathbf{E}_{tr}^T \mathbf{N}^{-1/2} (\mathbf{y}^o - \mathbf{y}^m)$	error cov, \mathbf{I}_{tr}
radiance climatology:	$\mathbf{0}$	covariance, $\mathbf{\Lambda}_{tr} - \mathbf{I}_{tr}$

Optimal estimation of radiances (2)

observation: $\mathbf{E}_{tr}^T \mathbf{N}^{-1/2} (\mathbf{y}^o - \mathbf{y}^m)$ error cov, \mathbf{I}_{tr}

radiance climatology: $\mathbf{0}$ covariance, $\mathbf{\Lambda}_{tr} - \mathbf{I}_{tr}$

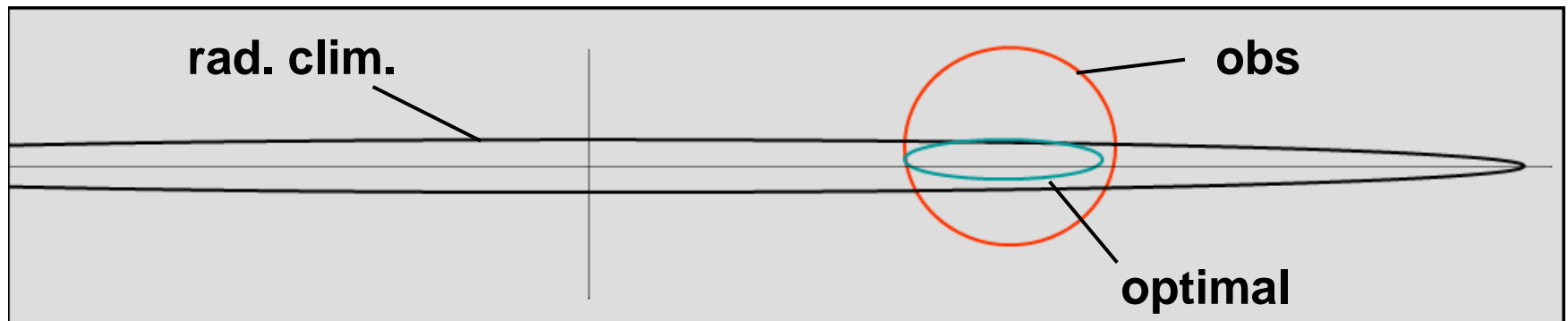
→ optimal estimate: \mathbf{y}_{tr}' error cov, \mathbf{Q}_{tr}

- $\mathbf{y}_{tr}' = \mathbf{Q} \mathbf{E}_{tr}^T \mathbf{N}^{-1/2} (\mathbf{y}^o - \mathbf{y}^m)$

- $\mathbf{Q}_{tr} = (\mathbf{\Lambda}_{tr} - \mathbf{I}_{tr}) \mathbf{\Lambda}_{tr}^{-1}$

$q_i = (\lambda_i - 1) / \lambda_i = \text{“pc weight”}$

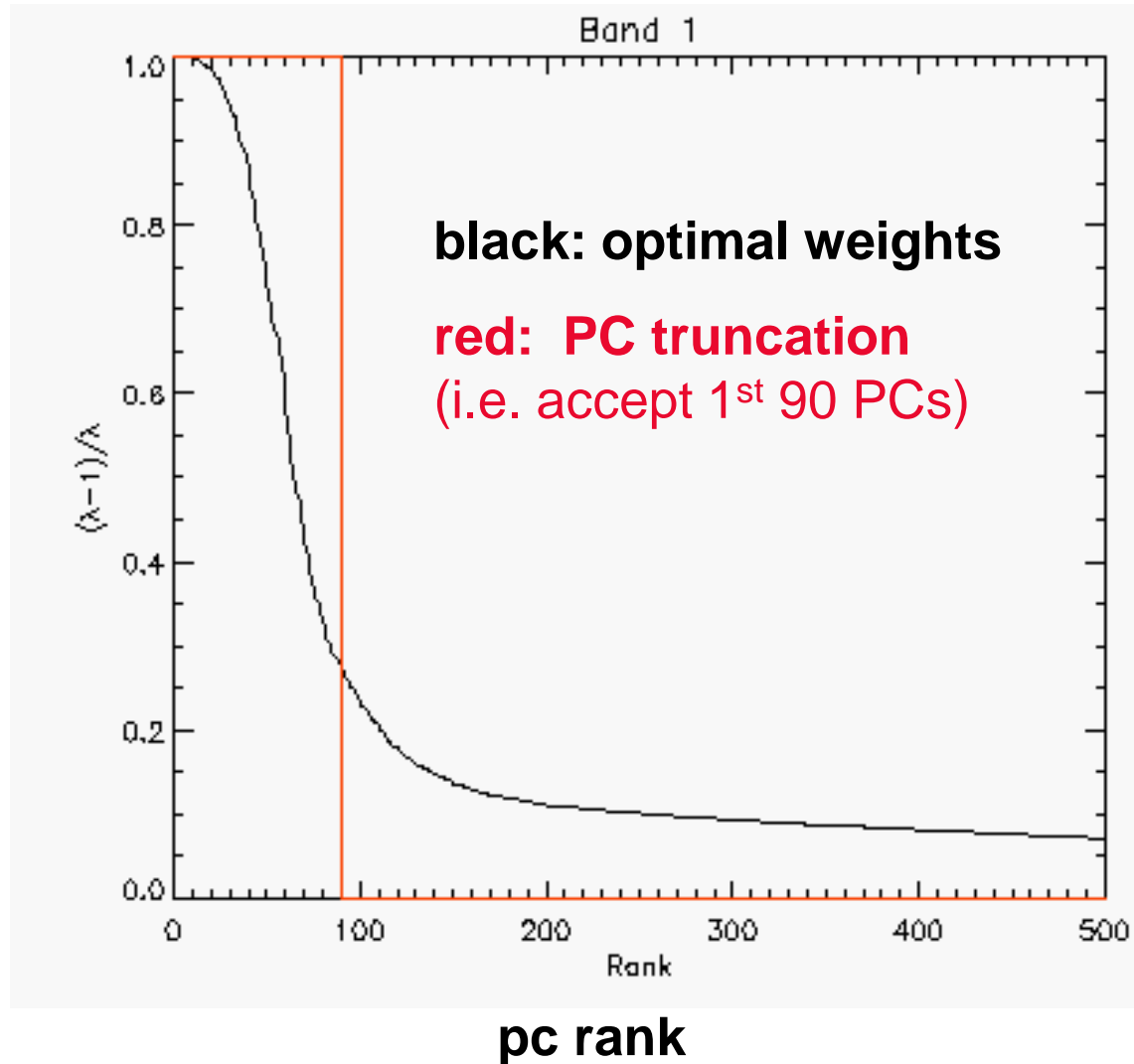
Note: normal “PC truncation” just gives “observation”



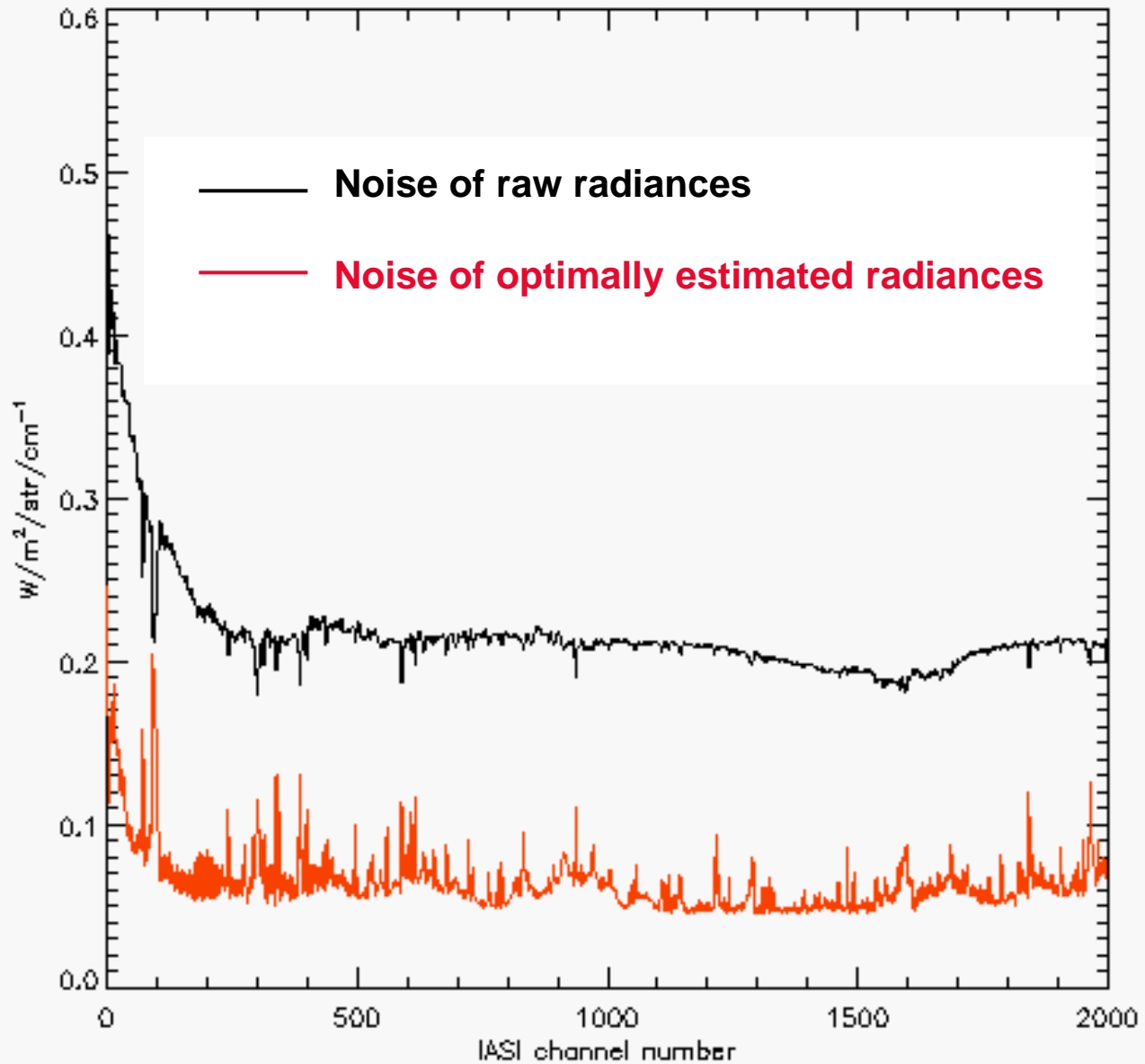
Relationship to PC truncation: IASI band 1

pc weight:

$$q = (\lambda - 1) / \lambda$$



Noise reduction – IASI band 1

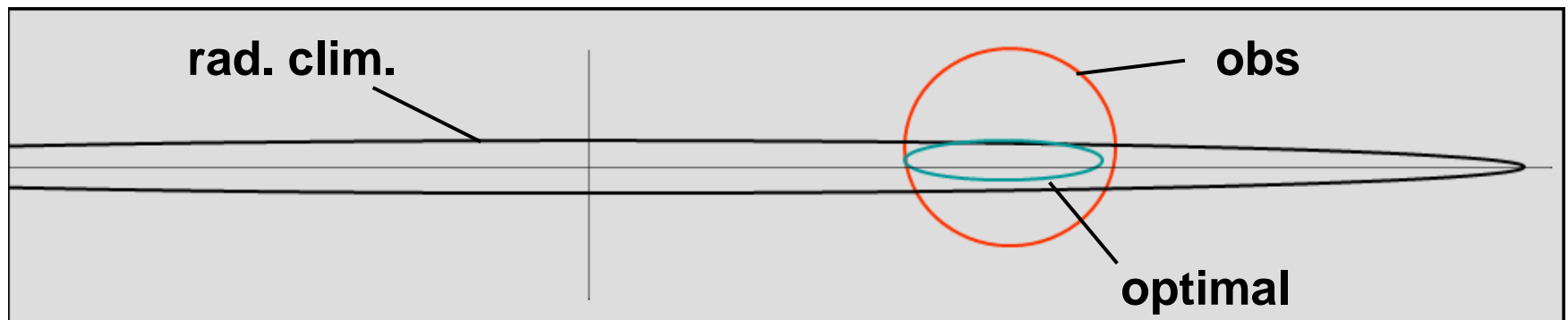


Optimal estimation of radiances (3)

Conclusions:

- Optimal radiance estimation gives **large** reductions in noise
- ... **but** noise is removed mainly in “directions” containing no atmospheric information
- ... and so it is **not** expected to lead to large reductions in retrieval error.

- But does it lead to **any** improvement?





Retrieval error analysis (linear)

radiances: $\mathbf{Q} \mathbf{E}_{\text{tr}}^T \mathbf{N}^{-1/2} (\mathbf{y}^o - \mathbf{y}^m)$

error cov: $\mathbf{Q} = (\mathbf{\Lambda}_{\text{tr}} - \mathbf{I}_{\text{tr}}) \mathbf{\Lambda}_{\text{tr}}^{-1}$

background: \mathbf{x}_b

error cov: \mathbf{B}

analysis: \mathbf{x}_a

error cov: \mathbf{A}

$$\mathbf{H} = \nabla_{\mathbf{x}} \mathbf{y}(\mathbf{x}) \quad ; \quad \mathbf{H}_{\text{PC}} = \mathbf{E}^T \mathbf{N}^{-1/2} \mathbf{H}$$

$$\mathbf{A}^{-1} = \mathbf{B}^{-1} + \mathbf{H}^T \mathbf{N}^{-1} \mathbf{H} = \mathbf{B}^{-1} + \mathbf{H}_{\text{PC}}^T \mathbf{I}^{-1} \mathbf{H}_{\text{PC}}$$

$$\mathbf{A}^{-1} = \mathbf{B}^{-1} + (\mathbf{H}_{\text{PC}}^T)_{\text{tr}} \mathbf{I}_{\text{tr}}^{-1} (\mathbf{H}_{\text{PC}})_{\text{tr}}$$

$$\mathbf{A}^{-1} = \mathbf{B}^{-1} + (\mathbf{H}_{\text{PC}}^T)_{\text{tr}} \mathbf{Q}_{\text{tr}}^{-1} (\mathbf{H}_{\text{PC}})_{\text{tr}}$$

all PCs, all channels

PC truncation

optimal

[Note assumption here: forward model error = 0]

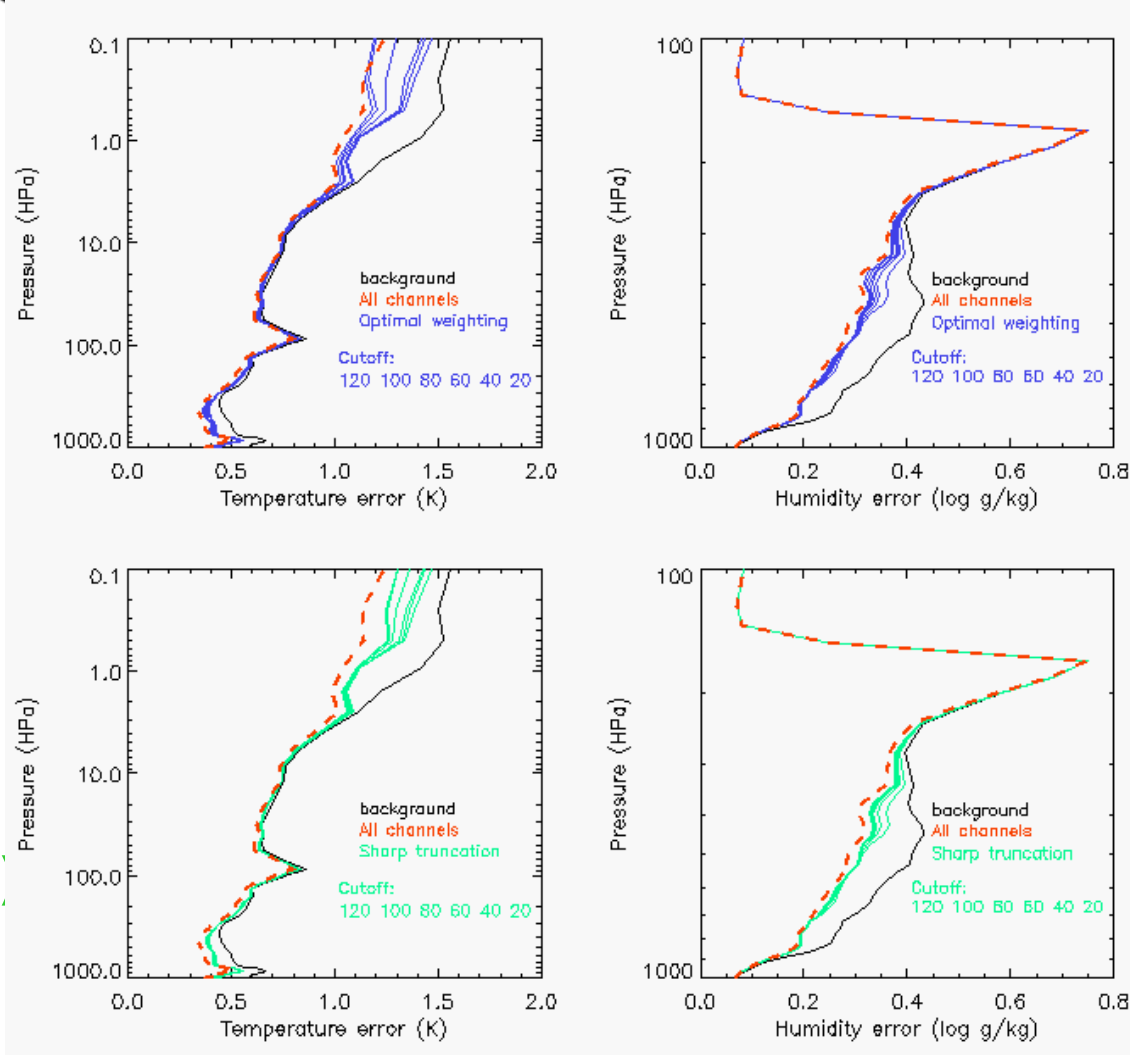
Retrieval errors – IASI band 1 (1)

PRELIMINARY

**optimal
weighting**

temperature

ln(spec.hum)



black: prior error

red: all PCs

**blue: optimal, at
various
truncations**

**green: PCs, at
same
truncations**

**truncations: 20,
40, 60, 80, 100,
120 PCs**

**PCs
(normal
truncation)**



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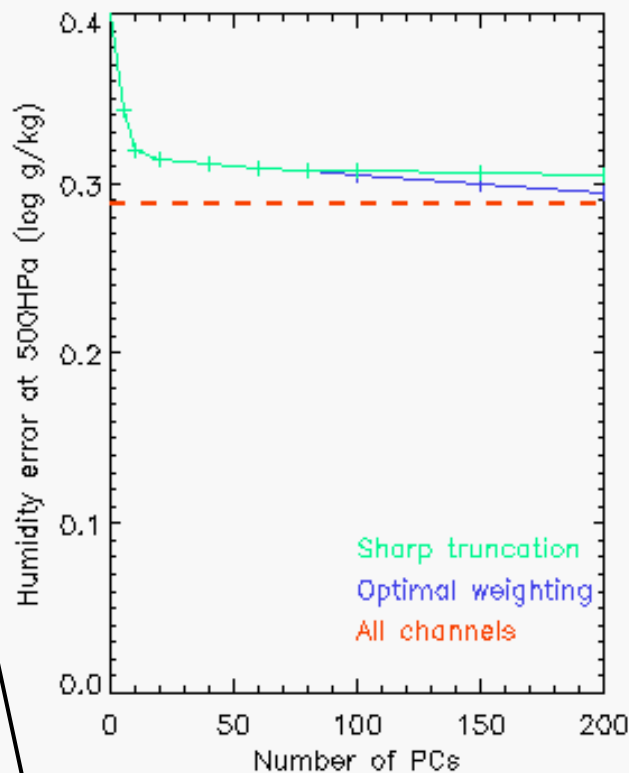
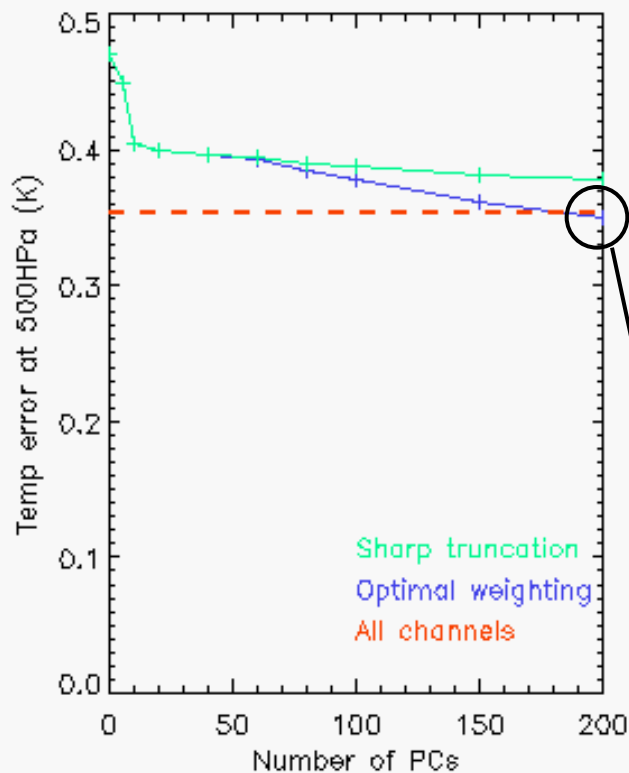
Retrieval errors – IASI band 1 (2)

PRELIMINARY

temperature

500 hPa

In(spec.hum)



— all channels

— PCs

— optimal

Note: No forward model error !!

better than all channels?



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Retrieval errors - results so far

- PC truncation – as expected:
 - Increasing the number of PCs gives decreasing retrieval error
 - ... but gain is small above a relatively low number of PCs
 - Retrieval error never reduces below the level for all channels / all PCs
- Optimal radiance estimation:
 - Theory suggests that, for a given PC truncation, optimally estimated radiances can give lower retrieval error than PC truncation
 - Results confirm this in the range 50-150 PCs for IASI band 1
 - **Caution.** At large number of PCs, as $q_i \rightarrow 0$, results become unstable, through mismatches between RTTOV Jacobians for IASI and IASI PCs computed from real data ([acknowledgment.: A.Collard](#))



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Conclusions

- PC of radiance climatologies have demonstrated “**engineering**” advantages
- ...but do they have additional **scientific** advantages, in terms of reduced retrieval error?
- Expected conclusions (when this study is finished):
 - PC truncation gives higher retrieval error than all channels
 - ...but information gain is small above a well-chosen truncation
 - Optimal radiance estimation can give lower retrieval error than all channels
 - ...but insignificantly lower??? – more work needed



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Thank you! Questions?

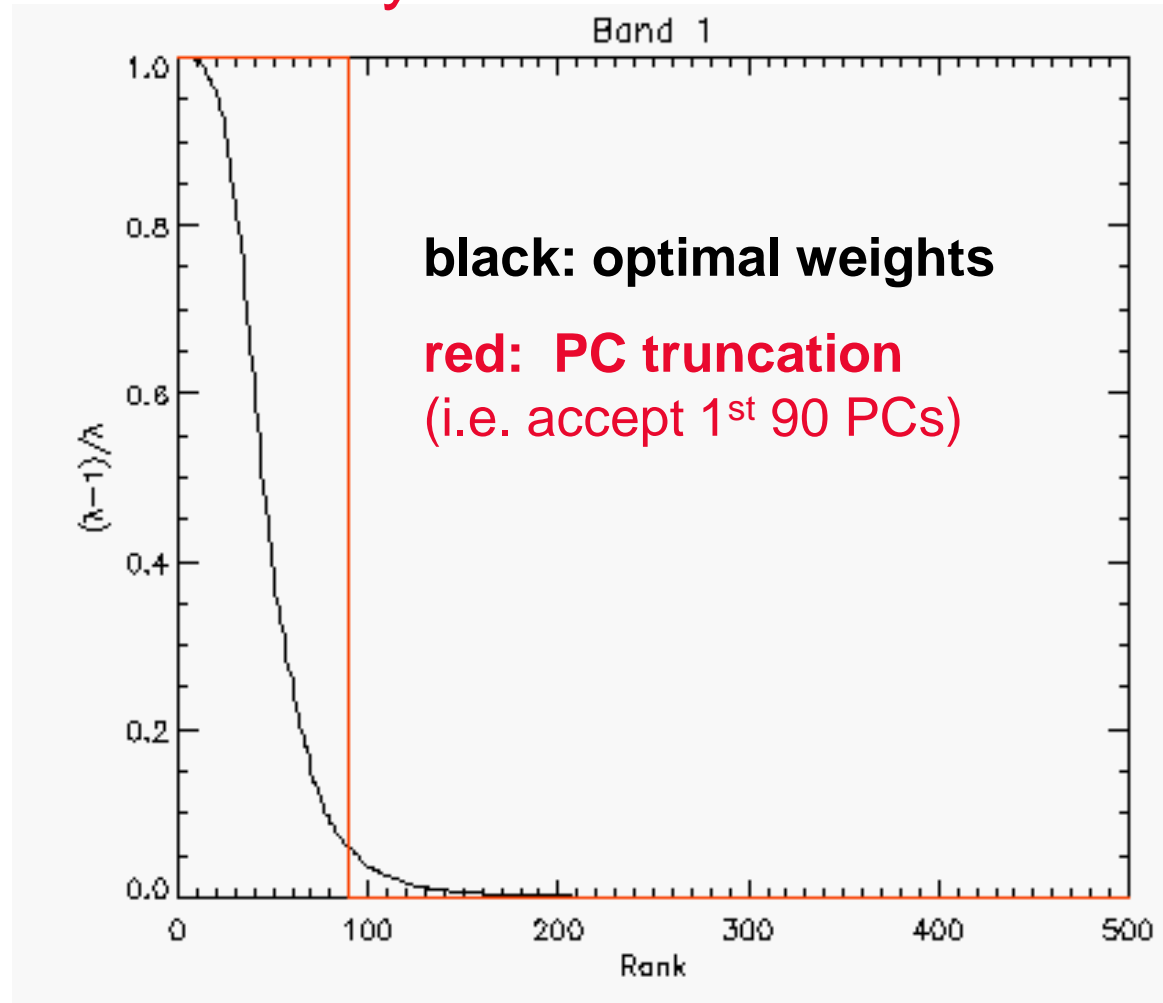


Relationship to PC truncation: IASI band 1

PCs from synthetic cloud-free data

pc weight:

$$q = (\lambda - 1) / \lambda$$



pc rank



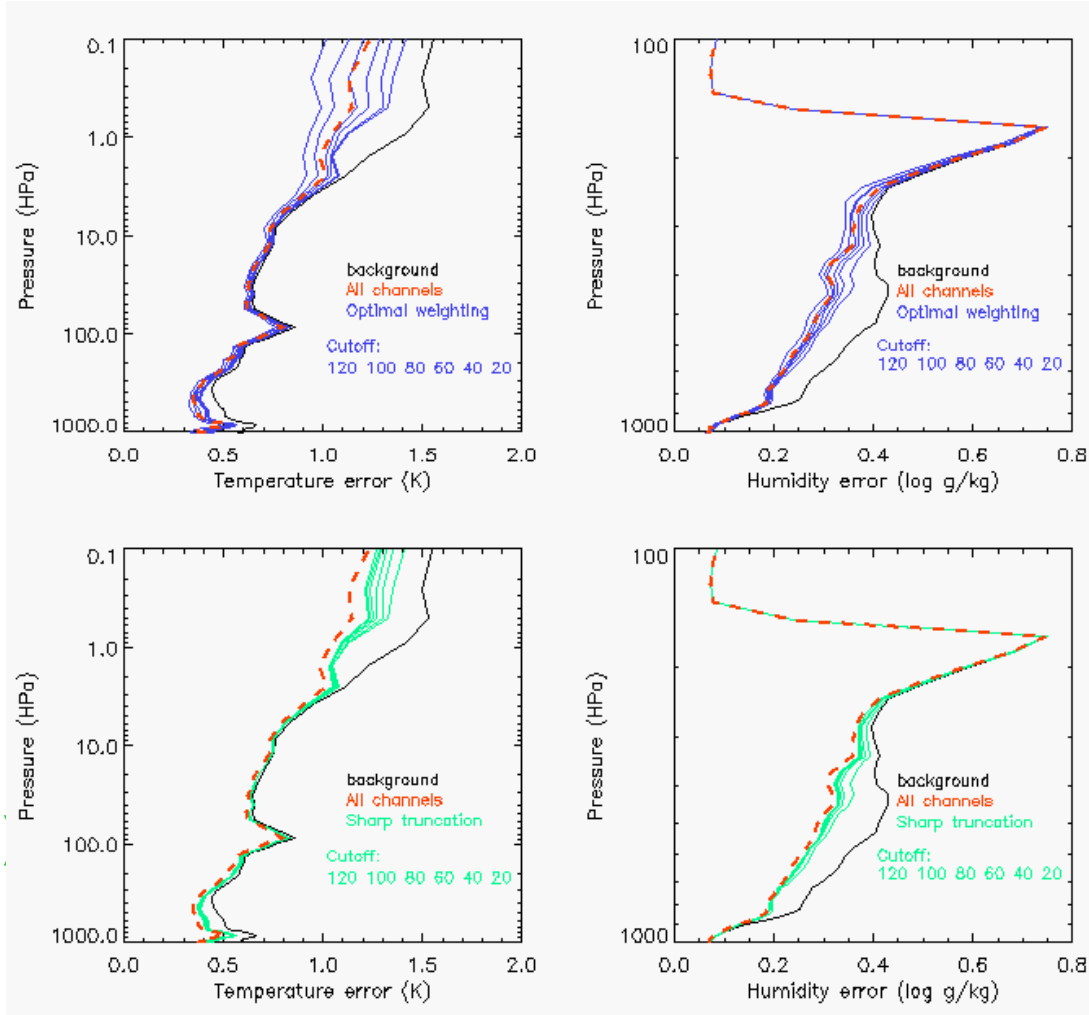
Retrieval errors – IASI band 1 (1)

PCs from synthetic cloud-free data

T

ln(q)

optimal weighting



black: prior error

red: all PCs

blue: optimal, at various truncations

green: PCs, at same truncations

truncations: 20, 40, 60, 80, 100, 120 PCs

PCs (normal truncation)



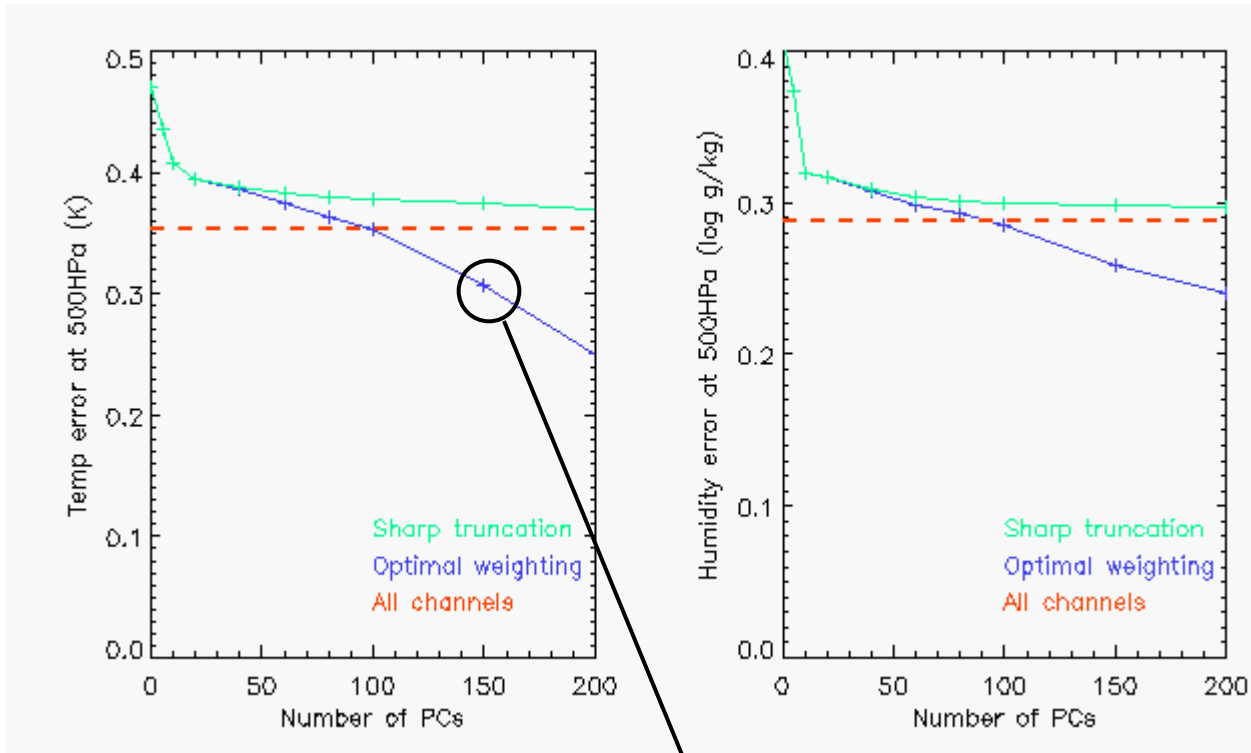
Retrieval errors – IASI band 1 (2)

PCs from synthetic cloud-free data

T

500 hPa

ln(q)



— all channels

— PCs

— optimal

Note: No forward model error !!

unstable calculation?



A Bayesian perspective (1)

Conditional probability of state \mathbf{x} given observations \mathbf{y}^o (Rodgers 1976):

$$P(\mathbf{x}|\mathbf{y}^o) = P(\mathbf{y}^o|\mathbf{x}) P(\mathbf{x}) / P(\mathbf{y}^o)$$

To find most probable \mathbf{x} :

- maximise $P(\mathbf{x}|\mathbf{y}^o)$ with respect to \mathbf{x} ...
- equivalent to maximising $\ln\{P(\mathbf{x}|\mathbf{y}^o)\}$...
- giving usual “cost function” $J(\mathbf{x})$ used in variational analysis (Lorenz 1988):

$$J(\mathbf{x}) = k_1 - \ln\{P(\mathbf{x}|\mathbf{y}^o)\} = k_1 - \ln\{P(\mathbf{y}^o|\mathbf{x})\} - \ln\{P(\mathbf{x})\} + \ln\{P(\mathbf{y}^o)\}$$

$$(\mathbf{y}^o - \mathbf{y}(\mathbf{x}))^T \mathbf{R}^{-1} (\mathbf{y}^o - \mathbf{y}(\mathbf{x})) \quad (\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}_b) \dots$$

...if PDFs are Gaussian \rightarrow cost function takes normal quadratic form.



A Bayesian perspective (2)

$$J(\mathbf{x}) = k_1 - \ln\{P(\mathbf{x}|\mathbf{y}^o)\} = k_1 - \ln\{P(\mathbf{y}^o|\mathbf{x})\} - \ln\{P(\mathbf{x})\} + \ln\{P(\mathbf{y}^o)\}$$

- The final term is usually absorbed into the constant and ignored
- ...because $P(\mathbf{y}^o)$ usually assumed constant (over the range of physically possible observed values)
- If we have prior knowledge of \mathbf{y}^o through a radiance climatology, we can replace the constant $P(\mathbf{y}^o)$ by a Gaussian of the form:

$$\ln\{P(\mathbf{y}^o)\} = k_2 - (\mathbf{y}^o - \mathbf{y}^m)^T \cdot (\mathbf{C}_o)^{-1} \cdot (\mathbf{y}^o - \mathbf{y}^m)$$

where \mathbf{y}^m and \mathbf{C}_o are the mean and covariance of the radiance climatology

- This clearly changes the form of the cost function, and hence the value of $J(\mathbf{x})$ for a given \mathbf{y}^o .



A Bayesian perspective (3)

HOWEVER

- $\ln\{P(y^o)\}$ is **not** a function of $x \rightarrow$
- when we differentiate

$$J(\mathbf{x}) = k_1 - \ln\{P(\mathbf{x}|\mathbf{y}^o)\} = k_1 - \ln\{P(\mathbf{y}^o|\mathbf{x})\} - \ln\{P(\mathbf{x})\} + \ln\{P(\mathbf{y}^o)\},$$

with respect to \mathbf{x} , then $\ln\{P(\mathbf{y}^o)\}$ disappears !!!

- Therefore, it can have no effect on the optimal value of x .

Therefore, from this perspective - i.e. radiance climatology as prior information on \mathbf{y}^o - it does not allow us to improve our estimate of x .



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