Accounting for Correlated Satellite Observation Error in NAVGEM

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Sources of Observation Error

1) Instrument error (usually, but not always, uncorrelated)
2) Mapping operator (**H**) error (interpolation, radiative transfer)
3) Pre-processing, quality control, and bias correction errors
4) Error of representation (sampling or scaling error), which can lead to correlated error:

True Temperature in Model Space

Subgrid Scale Temperature

T=28°	T=38°	T=58°
T=30°	T=44°	T=61°
T=32°	T=53°	T=63°

Current Practice

- Until recently, most operation DA systems assumed no correlations between observations at different levels or locations (i.e., a diagonal **R**)
- To compensate for observation errors that *are* actually correlated, one or more of the following is typically done:
  - Discard ("thin") observations until the remaining ones are uncorrelated (Bergman and Bonner (1976), Liu and Rabier (2003))
  - Local averaging ("superobbing") (Berger and Forsythe (2004))
  - Inflate the observation error variances (Stewart et al. (2008, 2013)
- Theoretical studies (e.g. Stewart et al., 2009) indicate that including even *approximate* correlation structures outperforms diagonal R with variance inflation

- *In January, 2013, the Met Office went operational with a vertical observation error covariance submatrix for the IASI instrument, which showed forecast benefit in seasonal testing in both hemispheres (Weston et al. (2014))

Methods to Estimate Covariance Matrices

- Several methods exist which can inform estimates of the background and/or observation error covariance matrices
- All methods have free parameters and make different assumptions; none are clearly superior to the others.
- Knowledge of when and how each method may produce sub-optimal results is the subject of current research.

1\. **Desroziers' Method**

(Desroziers *et al.* 2005)

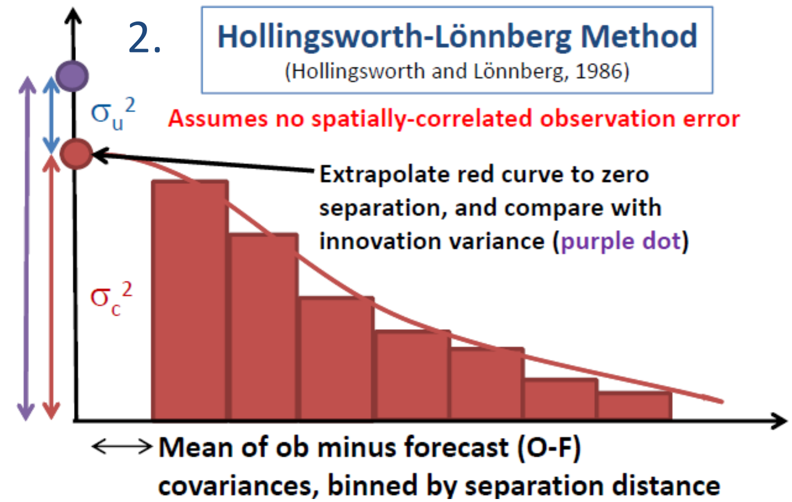
$$\left\langle (\vec{O}-\vec{F})(\vec{O}-\vec{A})^T \right\rangle = \mathbf{R}$$

$$\left\langle (\vec{A}-\vec{F})(\vec{O}-\vec{F})^T \right\rangle = \mathbf{HBH}^T$$

$$\left\langle (\vec{O}-\vec{F})(\vec{O}-\vec{F})^T \right\rangle = \mathbf{R}+\mathbf{HBH}^T$$

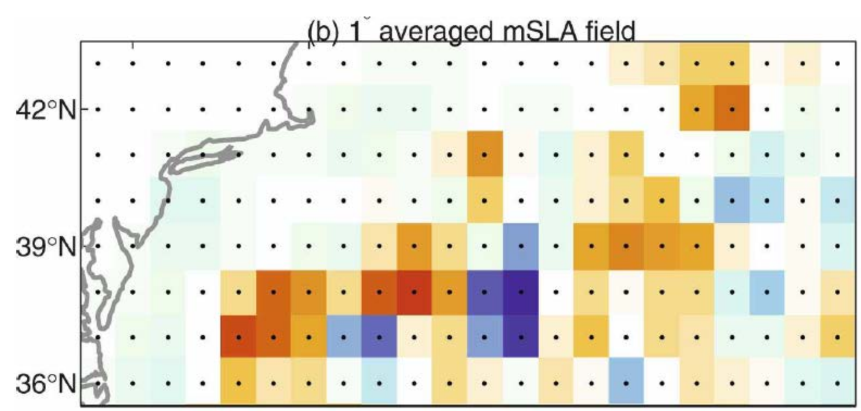
2\. **Hollingsworth-Lönnberg Method**

(Hollingsworth and Lönnberg, 1986)



3\. **Observation Based Methods**

e.g. Oke and Sakov 2007



4DVar Primal Formulation

$$\underline{w} \equiv \underline{x} - \underline{x_f} = BH^T \left(HBH^T + R\right)^{-1} \left(\underline{y} - H\underline{x_f}\right)$$

$$\left(B^{-1} + H^T R^{-1} H\right)\underline{w} = \left(B^{-1} + H^T R^{-1} H\right) BH^T \left(HBH^T + R\right)^{-1} \left(\underline{y} - H\underline{x_f}\right)$$

$$\left(B^{-1} + H^T R^{-1} H\right)\underline{w} = H^T R^{-1} \left(\underline{y} - H\underline{x_f}\right)$$

$$\underline{s} \equiv B^{1/2}\underline{w}$$

$$\underline{w} = B^{-1/2}\underline{s}$$

Preconditioning is done with $B^{-1/2}$

$$B^{-1/2}\left(B^{-1} + H^T R^{-1} H\right)\left(B^{-1/2}\underline{s}\right) = B^{-1/2} H^T R^{-1} \left(\underline{y} - H\underline{x_f}\right)$$

$$\boxed{\left(I + B^{-1/2} H^T R^{-1} H B^{-1/2}\right)\underline{s} = B^{-1/2} H^T R^{-1} \left(\underline{y} - H\underline{x_f}\right)}$$

Iteration is done on this problem. We need to invert R!

4DVar Dual Formulation

$$(HBH^T + \tilde{R})\underline{z} = (\underline{y} - H\underline{x_b})$$

$$\tilde{R}^{-1/2}(HBH^T + \tilde{R})\underline{z} = \tilde{R}^{-1/2}(\underline{y} - H\underline{x_b})$$

Change of variables

$$\underline{w} = \tilde{R}^{1/2}\underline{z}$$

$$\underline{z} = \tilde{R}^{-1/2}\underline{w}$$

$$\boxed{\begin{aligned} R &= \tilde{R}^{1/2} C \tilde{R}^{1/2} \\ \tilde{R} &= diag\{\sigma_{i,j}\} \end{aligned}}$$

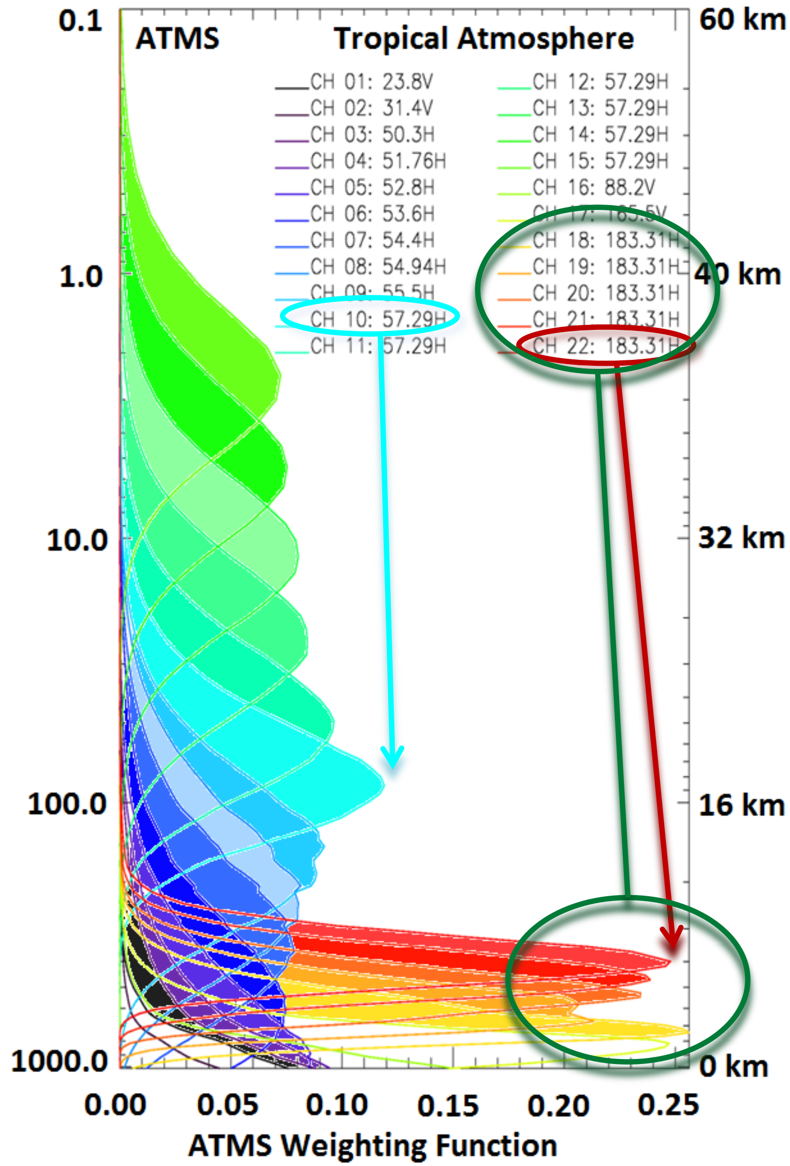
$$\tilde{R}^{-1/2}(HBH^T + \tilde{R})\tilde{R}^{-1/2}\left(\tilde{R}^{1/2}\underline{z}\right) = \tilde{R}^{-1/2}(\underline{y} - H\underline{x_b})$$

$$\boxed{(\tilde{R}^{-1/2}HBH^T\tilde{R}^{-1/2} + I)\underline{w} = \tilde{R}^{-1/2}(\underline{y} - H\underline{x_b})}$$

C

Iteration is done on the partial step and then mapped back with BH^T

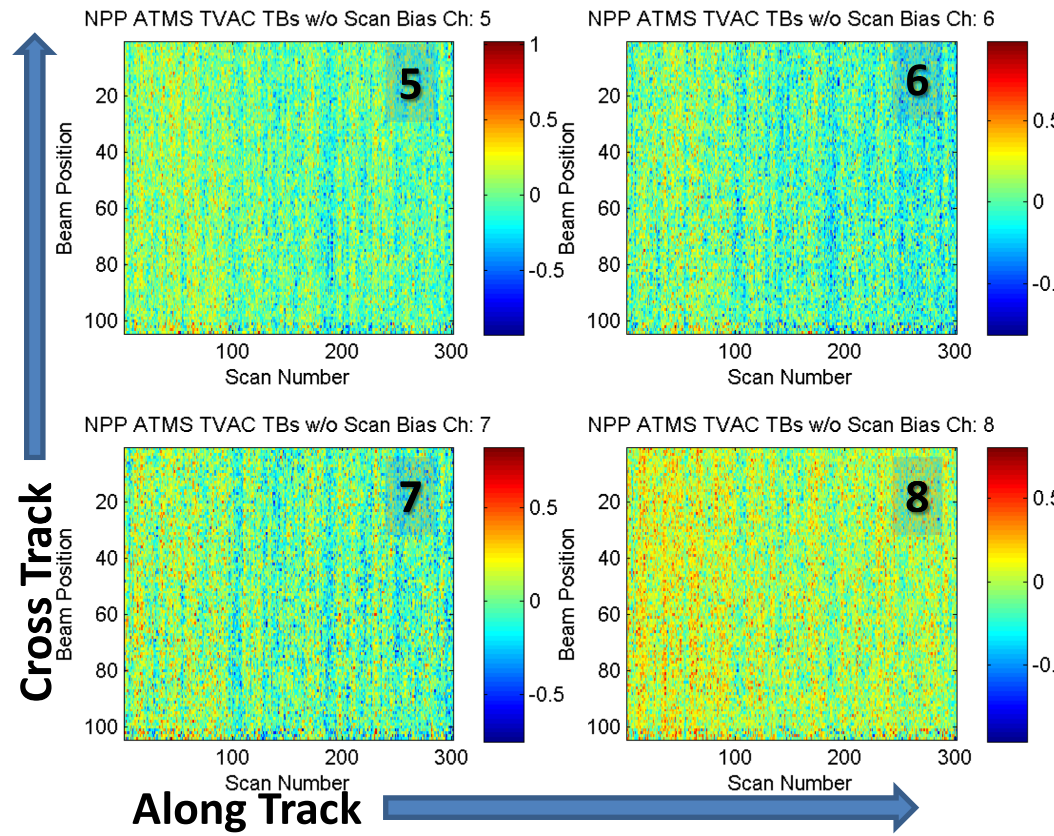
Application to ATMS



Advanced Technology Microwave Sounder (ATMS)

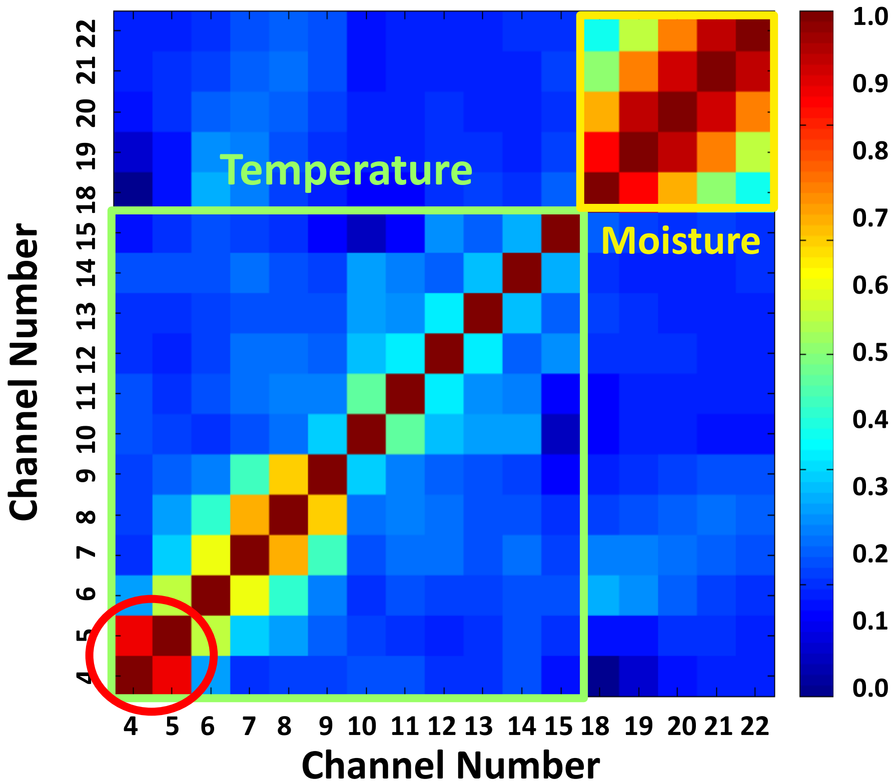
13 temperature channels

9 moisture channels



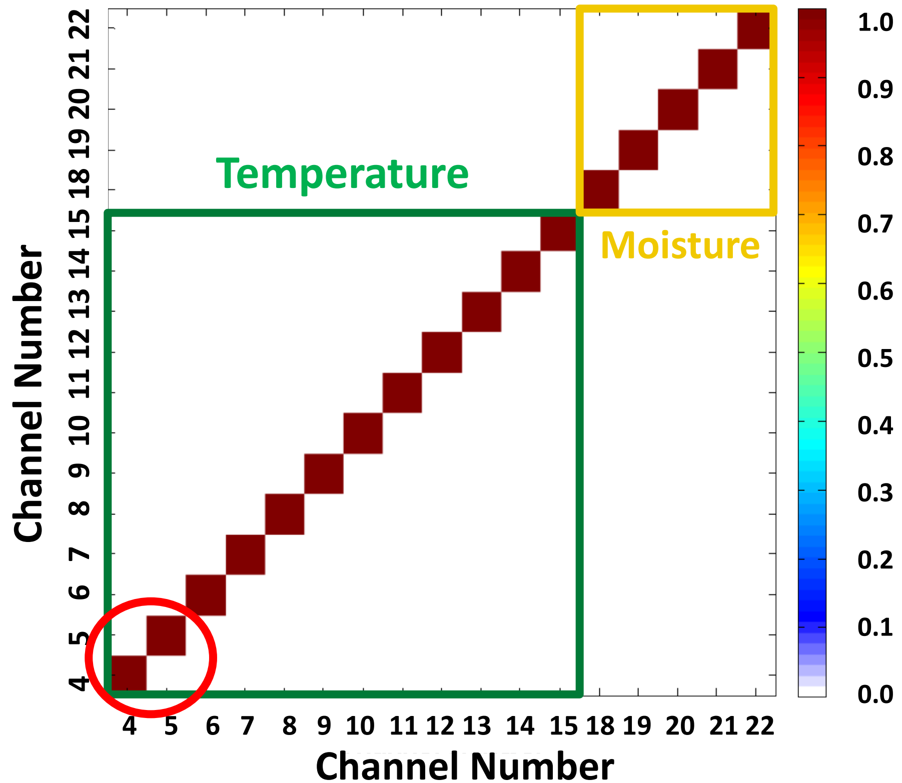
Observation Error Correlation Estimation for ATMS

Statistical Estimate



Desroziers' method estimate of interchannel portion of observation error correlation matrix for ATMS

Current Treatment

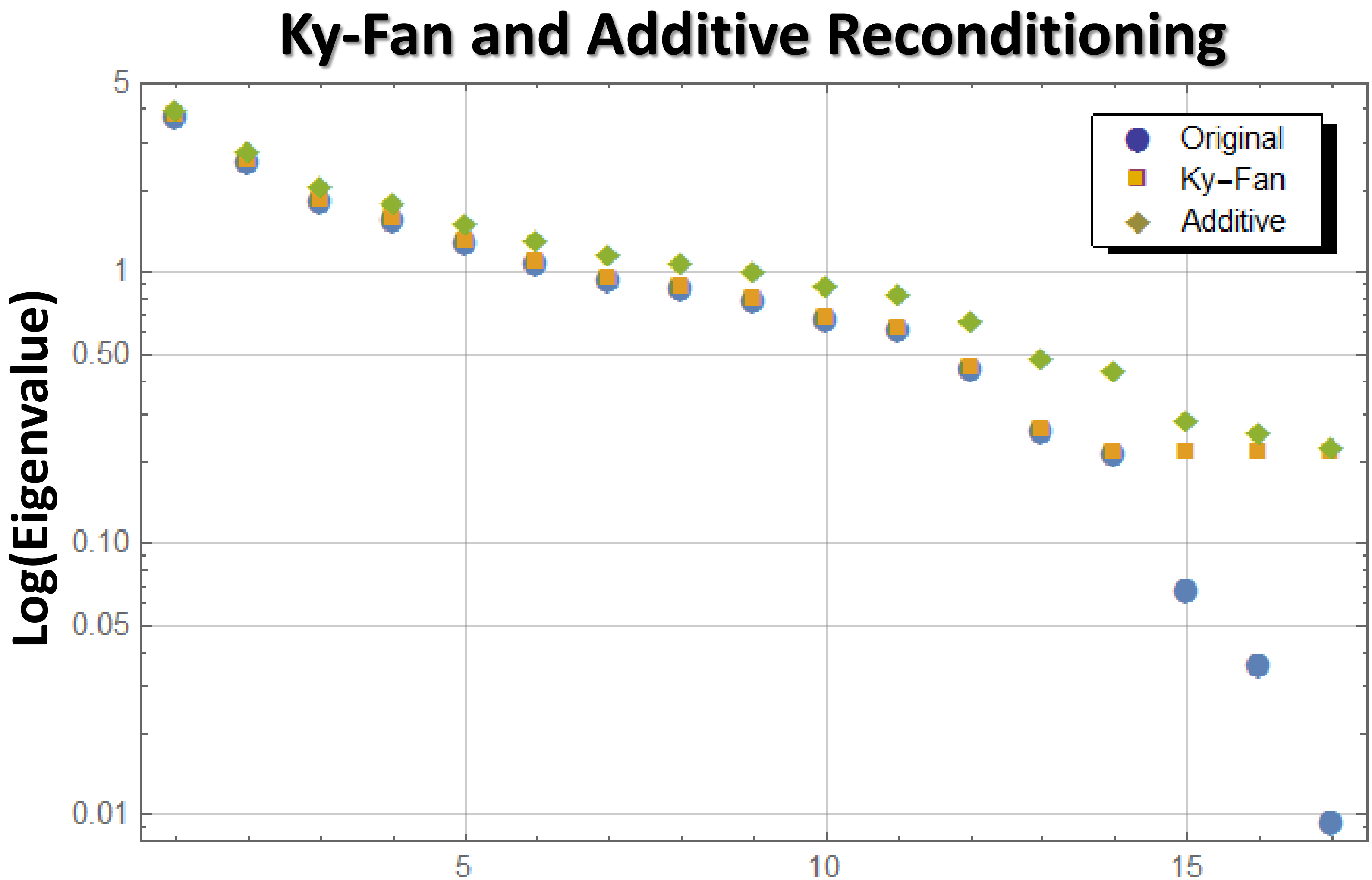


Current observation error correlation matrix used for ATMS, and for *ALL observations*

Practical Implementation: What about Convergence?

- The condition number of a matrix X is defined by $\sigma_{max}(X)/\sigma_{min}(X)$, which is the ratio of the maximum singular value of X to the minimum one. (Singular value == eigenvalue for symmetric X)
- Adding correlated error increases the condition number, slowing down convergence of the solver.
- We can control how long the solver takes by constructing an approximate matrix with *any condition number we choose*.
- How to improve conditioning:
  1. Preconditioning by multiplying by diagonal scaling matrices
  2. Increase the diagonal values (additively) of the matrix (e.g. Weston et al. (2014)).
  3. Find a positive definite approximation to the matrix by altering the eigenvalue spectrum (Ky-Fan p-k norm).

ATMS Original and Reconditioned Eigenspectra



Practical Implementation: Cauchy Interlacing Theorem

What happens when radiance profiles are incomplete (i.e., at a given location, some channels are missing, usually due to failing QC checks)?

Cauchy interlacing theorem

Let A be a symmetric $n \times n$ matrix. The $m \times m$ matrix B, where $m \leq n$, is called a compression of A if there exists an orthogonal projection P onto a subspace of dimension m such that $P^*AP = B$. The Cauchy interlacing theorem states:

Theorem. If the eigenvalues of A are $\alpha_1 \leq \ldots \leq \alpha_n$, and those of B are $\beta_1 \leq \ldots \leq \beta_j \leq \ldots \leq \beta_m$, then for all $j < m + 1$,

$$\alpha^j \leq \beta^j \leq \alpha^{n-m+j}$$

Notice that, when $n - m = 1$, we have $\alpha_j \leq \beta_j \leq \alpha_{j+1}$, hence the name *interlacing* theorem.

Experimental Design

Experiment Name	95% CI	99% CI	99.99% CI	Mean Iter	Description
atid	0	0	0	56	Control run, no correlated error for ATMS or IASI, default diag(R)
atmsc018	5	3	3	68	Recondition Desroziers ATMS correlation matrix to 18, default diag(R)
iasic169	5	3	2	72	Recondition Desroziers IASI correlation matrix to 169, default diag(R)
atmsiasi	8	5	2	78	Both of the above
Dzratmsc018	9	5	3	81	Same as atmsc018, Desroziers diag(R), moisture 1/2 compromise diag(R)
Dzriasic169	4	3	2	88	Same as iasic169, Desroziers diag(R), moisture 1/2 compromise diag(R)
Drzatmsiasi	10	4	4	104	Same as atmsiasi, Desroziers diag(R), moisture 1/2 compromise diag(R)
Wesatmsc018	13	4	3	65	Same as Dzratmsc018, but uses Weston-style reconditioning
Wesiaisc169	12	9	2	84	Same as Dzriasic169, but uses Weston-style reconditioning
Wesboth	16	13	3	87	Same as Drzatmsiasi, but uses Weston-style reconditioning

ATMS only	IASI only	ATMS & IASI

Wesboth (Proposed Scorecard*)

2013070100 - 2013083118 Wesboth: Correlated error for both ATMS and IASI with Weston reconditioning

	Reference	Level	Region	Variable	Lead Time	Metric	Weight	Score
Buoy	Fixed Buoy	None	NH	Wind Speed	72	Mean Error	2	0
	Fixed Buoy	None	SH	Wind Speed	72	Mean Error	2	0
	Fixed Buoy	None	Tropics	Wind Speed	72	Mean Error	2	0
Raob	Radiosondes	100.0	Global	Geopotential	72	RMSE	1	1
	Radiosondes	250.0	Global	Air Temp	72	RMSE	1	0
	Radiosondes	250.0	Global	Wind	72	Vector RMSE	1	0
	Radiosondes	500.0	Global	Geopotential	72	RMSE	1	1
	Radiosondes	850.0	Global	Air Temp	72	RMSE	1	0
	Radiosondes	850.0	Global	Wind	72	Vector RMSE	1	1
ECMWF-Analysis	EC-Analysis	200.0	NH	Wind	72	Vector RMSE	1	1
	EC-Analysis	200.0	Tropics	Wind	72	Vector RMSE	1	1
	EC-Analysis	500.0	NH	Geopotential	96	AC	4	4
	EC-Analysis	500.0	SH	Geopotential	96	AC	1	0
	EC-Analysis	850.0	NH	Wind	72	Vector RMSE	1	1
	EC-Analysis	850.0	Tropics	Wind	72	Vector RMSE	2	2
	EC-Analysis	1000.0	NH	Geopotential	96	AC	1	0
	EC-Analysis	1000.0	SH	Geopotential	96	AC	1	1
								13

*Same as FNMOC standard scorecard, with self-analysis replaced by ECMWF analysis, confidence level from 95% to 99%, no thresholding

Main Conclusions

- **The Desroziers error covariance estimation methods can quantify correlated observation error**
- **Minimal changes can be made to the estimated error correlations to fit operational time constraints**
- **After accounting for correlations, reducing default variances improves forecasts**
- **Correctly accounting for correlated observation error in satellite data assimilation improves forecasts**
- **One must be careful comparing experiments using scorecards, especially those with thresholding.**