Robust quantification of uncertainty on short-range model forecasts in radiance space based on reference sonde data

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Introduction

Robust statistical estimations of numerical weather prediction model uncertainties are of crucial importance for maximizing the impact of the data that is operationally assimilated, which is mainly composed of satellite radiance measurements. One of the most direct ways to assess these errors is to compare short-range model forecasts with high-quality radiosonde data such as those from the GCOS Reference Upper-Air Network (GRUAN). The Met Office, as part of its activities within the Gap Analysis for Integrated Atmospheric ECV CLImate Monitoring (GAIA-CLIM) EU-funded project, have been developing the GRUAN Processor designed to compare simulated satellite measurements from model data against simulated radiances calculated from GRUAN sonde data through the use of the RTTOV fast radiative transfer model. The Processor also aims to quantify all known sources of uncertainties associated with sonde measurements so as to assess the statistical significance of their departures from simulated satellite soundings. To account for the total uncertainty on the departures, however, we also need to include contributions from forecast uncertainty and use of different vertical grids, as discussed here.
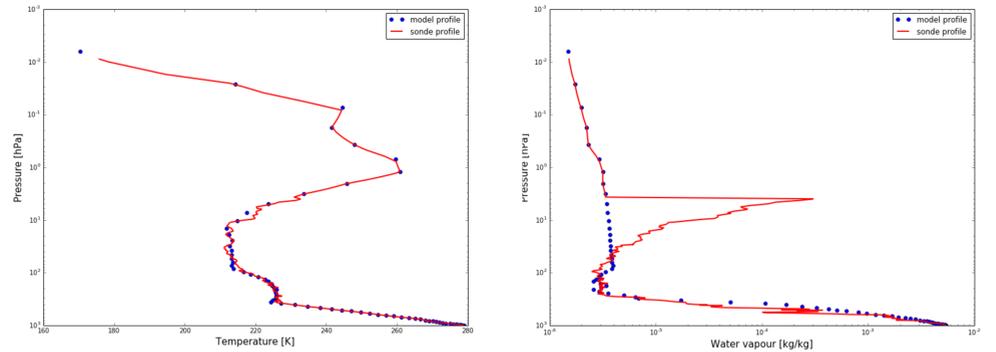

Figure 1 Sonde and model's forecast temperature (left panel) and humidity (right panel) profiles at Lindenberg on 9 March 2017 2200 UTC

Error estimation procedure

It is assumed that the sonde profile is on a "fine" grid with 278 levels and the model's forecast, generated with the Met Office Unified Model, is on a "coarse" grid with 70 levels (see Figure 1). Let $\mathbf{y}_m$ and $\mathbf{y}_s$ be the brightness temperature vectors generated by the RTTOV fast radiative transfer code when the $\mathbf{x}_m$ model profile interpolated on the fine grid and the $\mathbf{x}_s$ sonde profile are given as input, H the forward model (or observation) operator, $\mathbf{H}$ its derivative with respect to the state vector (or jacobian), $\boldsymbol{\varepsilon}$ a (random) error vector on the fine grid, $\boldsymbol{\delta}$ a (random) error vector on the coarse grid, $\mathbf{W}$ a linear interpolation matrix and $\boldsymbol{\varepsilon}_{\mathrm{intp}} = \mathbf{W}\mathbf{z}_t - \mathbf{x}_t$ the vertical interpolation error, where $\mathbf{x}_t$ and $\mathbf{z}_t$ are the true state vector on the fine and the coarse grid, respectively. We can then write

$$\mathbf{y}_s \equiv H(\mathbf{x}_s) \cong H(\mathbf{x}_t) + \mathbf{H}_{\mathbf{x}_t}\boldsymbol{\varepsilon}_s \qquad \mathbf{x}_m \equiv \mathbf{W}\mathbf{z}_m \qquad \mathbf{y}_m \equiv H(\mathbf{x}_m) = H(\mathbf{x}_t + \mathbf{W}(\mathbf{z}_m - \mathbf{z}_t) + \mathbf{W}\mathbf{z}_t - \mathbf{x}_t) \cong H(\mathbf{x}_t) + \mathbf{H}_{\mathbf{x}_t}\mathbf{W}\boldsymbol{\delta}_m + \mathbf{H}_{\mathbf{x}_t}\boldsymbol{\varepsilon}_{\mathrm{intp}} \qquad \delta\mathbf{y} \equiv \mathbf{y}_m - \mathbf{y}_s \cong \mathbf{H}_{\mathbf{x}_t}(\mathbf{W}\boldsymbol{\delta}_m + \boldsymbol{\varepsilon}_{\mathrm{intp}} - \boldsymbol{\varepsilon}_s)$$

Our goal is to estimate $\mathbf{S}_{\delta y}$, the total error covariance of $\delta\mathbf{y}$, given by $\mathbf{S}_{\delta y} \equiv \mathrm{cov}(\delta\mathbf{y}) = \mathbf{H}\mathbf{W}\mathbf{B}_m\mathbf{W}^T\mathbf{H}^T + \mathbf{H}\mathbf{R}_s\mathbf{H}^T + \mathbf{H}\mathbf{S}_{\mathrm{intp}}\mathbf{H}^T$

where $\mathbf{B}_m$ is the forecast error covariance on the coarse grid, $\mathbf{R}_m$ is the sonde error covariance on the fine grid and $\mathbf{S}_{\mathrm{intp}}$ is the vertical interpolation error covariance, discussed below.

Estimating forecast errors on finer grids

A "natural" guess for $\mathbf{B}_f$ is $\mathbf{B}_f \cong \mathbf{W}\mathbf{B}_c\mathbf{W}_c^T$ but this leads to a singular matrix. It is, however, possible to re-condition it by imposing that its correlation matrix has a unit matrix in the diagonal, as this is not guaranteed by the interpolation. Figure 2 shows $\mathbf{C}_c$ the temperature forecast error correlation matrix on model levels and on sonde levels after reconditioning, as well as the positive eigenvalues of these matrices. The vertical profile of temperature forecast error standard deviations on the sonde grid (Figure 3) is determined from the interpolation of that on the model grid, as $\boldsymbol{\sigma}_f = \mathbf{W}\boldsymbol{\sigma}_c$. Finally, we get $\mathbf{B}_f \equiv diag(\boldsymbol{\sigma}_f)\mathbf{C}_f^{recon}diag(\boldsymbol{\sigma}_f)$

Interpolation error

To estimate $\mathbf{S}_{\mathrm{intp}}$, the covariance of interpolation error (see Figure 3) we need an estimate $\hat{\mathbf{z}}_t$ of $\mathbf{z}_t$ given $\mathbf{x}_t$. We can use $\mathbf{W}^*$, the Moore-Penrose pseudo-inverse of $\mathbf{W}$ (see Rodgers, 2000, his section 10.3), given by
$$\hat{\mathbf{z}}_t = \mathbf{W}^*\mathbf{x}_t \equiv (\mathbf{W}^T\mathbf{B}_f^{-1}\mathbf{W})^{-1}\mathbf{W}^T\mathbf{B}_f^{-1}\mathbf{x}_t$$
where $\mathbf{B}_f$ is the forecast error covariance on the fine grid estimated as discussed above. Note that $\hat{\mathbf{z}}_t$ is a "consistent" estimate of $\mathbf{z}_t$ as we get $\hat{\mathbf{z}}_t = \mathbf{z}_t$ when $\mathbf{x}_t = \mathbf{W}\mathbf{z}_t$.

In this way we can write $\boldsymbol{\varepsilon}_{\mathrm{intp}} \equiv \mathbf{W}\mathbf{z}_t - \mathbf{x}_t = (\mathbf{W}\mathbf{W}^* - \mathbf{I})\mathbf{x}_t$
so that $\mathbf{S}_{\mathrm{intp}} \equiv \mathrm{cov}(\boldsymbol{\varepsilon}_{\mathrm{intp}}) = (\mathbf{W}\mathbf{W}^* - \mathbf{I})\mathbf{B}_f(\mathbf{W}\mathbf{W}^* - \mathbf{I})^T = \mathbf{B}_f(\mathbf{I} - \mathbf{W}\mathbf{W}^*)$
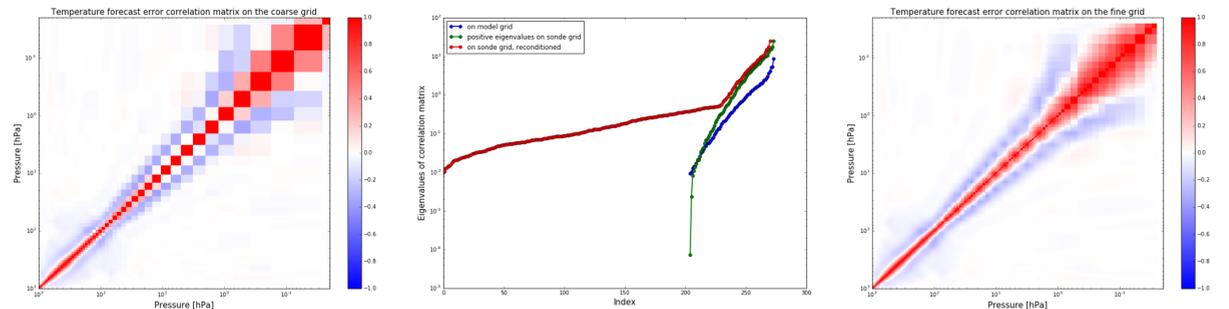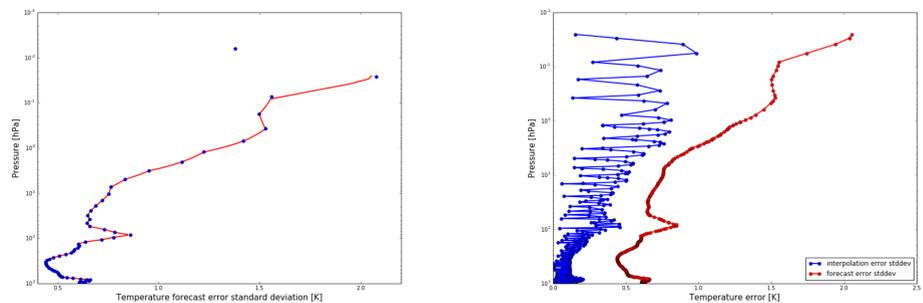

Figure 2 Temperature forecast error correlation matrix on model levels (or coarse grid, left panel); Eigenvalues of the temperature forecast error correlation matrix (middle panel) on the coarse grid (blue circles), on the fine grid after interpolation from the coarse grid (positive eigenvalues only, green circles) and on the fine grid after reconditioning (red circles); Temperature forecast error correlation matrix on the fine grid, after reconditioning (right panel).

Figure 3 Temperature forecast error standard deviation profile on the model grid (blue circles) and interpolated on the sonde grid (red solid line, left panel); Temperature interpolation error standard deviation profile on the sonde grid (blue solid line) and (for ease of comparison) temperature forecast error standard deviation profile on the sonde grid (right panel)
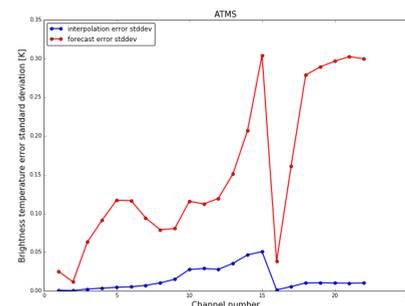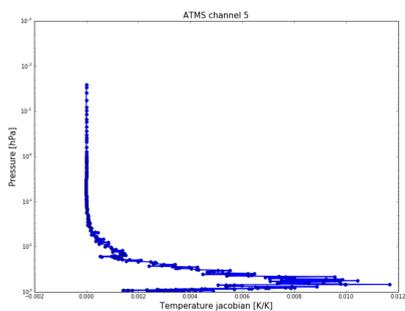

Errors in observation space

The interpolation error covariance in observation space is then calculated as $\mathbf{H}\mathbf{S}_{\mathrm{intp}}\mathbf{H}^T$. If we assume that forecast errors of the components of the state vector are mutually uncorrelated, total uncertainty is the sum of the uncertainty of the single components. Figure 4 shows the temperature jacobian for ATMS channel 5 and the resulting interpolation error for temperature. As hoped, the interpolation error for temperature has magnitude that is much smaller than that of temperature forecast error in radiance space.

Summary and future work

- We have described a robust procedure to assess forecast error uncertainty using reference sonde profiles
- The analysis performed on temperature errors needs to be repeated for humidity and surface parameter errors
- Total uncertainty is sum of single parts, assuming forecast errors of components are mutually uncorrelated
- Accurate specification of sonde errors also needs to be added to the total error budget
- Validity of used forecast errors can then be tested using an ensemble of forecast minus sonde profile departures

References

C. D. Rodgers (2000), Inverse methods for atmospheric sounding: theory and practice, World scientific

Figure 4 Temperature jacobian for channel 5 (52.8 GHz, same as AMSU-A channel 4) of the Advanced Technology Microwave Sounder (ATMS, top panel); temperature component of the interpolation error standard deviation [K] (blue solid line) and temperature forecast error in radiance space [K] (bottom panel) for ATMS channels.