Jacobian mapping between vertical coordinate systems in data assimilation
(ITSC-14 RTSP-WG action 2.1.1-c)

Atmospheric Science and Technology Directorate

Yves J. Rochon, Louis Garand, D.S. Turner, and Saroja Polavarapu

with contributions from Jacques Hallé, Shuzhan Ren, Yulia Nezlin

Content

- Introduction
- Interpolators
- Mapping comparisons
- 1D assimilation
- 3D-Var assimilation
- Summary and comments

Introduction

Context:

- ✓ Fast RTMs for assimilation of radiances from nadir sounders often rely on regression based models evaluated on fixed pressure levels (e.g. RTTOV).
- ✓ Numerical prediction (e.g. NWP) models often use different vertical levels and a different vertical coordinate (e.g. η-hybrid).
- ✓ In this circumstance, Jacobian mapping from RTM to model coordinate is required in data assimilation (DA).

✓ Data assimilation requires explicit pairing of the vertical interpolator and Jacobian mapping.

a) profile x' on RTM levels ← profile x on model levels

$$\mathrm{x}'(p_i) = x'_i = \mathrm{s}_i(\mathbf{x}) = \sum_j W_{i,j} x_j \qquad \text{or} \qquad \mathbf{x}' = \mathbf{W}\mathbf{x}$$

b) Jacobian mapping:

model vertical coordinate ← RTM vertical coordinate

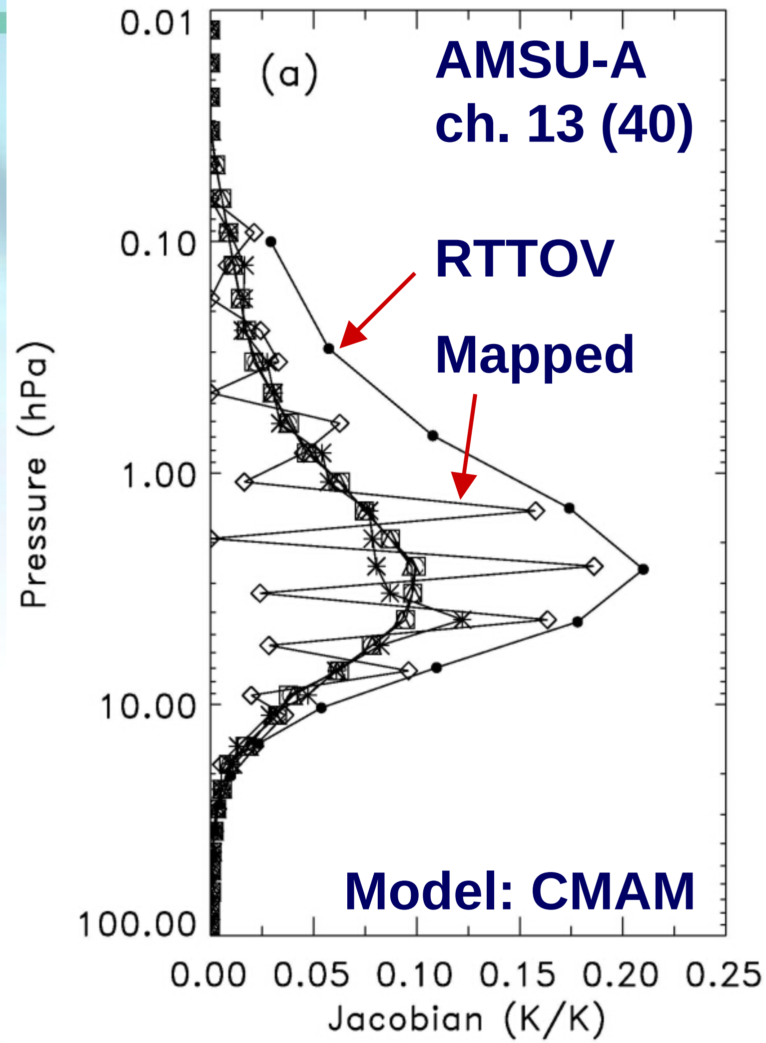
$$\left.\frac{\partial \mathrm{f}}{\partial x_j}\right|_{\mathbf{x}} = \sum_i \left.\frac{\partial \mathrm{f}}{\partial x'_i}\right|_{\mathbf{x}'} \frac{\partial x'_i}{\partial x_j} = \sum_i \left.\frac{\partial \mathrm{f}}{\partial x'_i}\right|_{\mathbf{x}'} W_{i,j} \qquad \text{or} \qquad \mathbf{h} = \mathbf{W}^T \mathbf{h}'$$

The Jacobian mapping matrix is the adjoint $\mathbf{W}^T$ of a linear forward model vertical interpolator matrix $\mathbf{W}$ (or TLM of the interpolator)

Introduction

Identification of problem:

- ✓ Model levels not participating in forward interpolation (blind levels) lead to improper Jacobian mapping.
- ✓ Blind levels can result when the model vert. resolution is sufficiently higher than the RTM vert. resolution.
- ✓ Improper mapping **heavily masked** by vert. correlations of background covariances.



Introduction

Remainder of presentation:

- ✓ Identify an appropriate design for the vertical interpolator and its adjoint for use with fast RTMs in data assimilation when required (part 2 of ITSC-14 RTSC-WG action 2.1.1-c)
- ✓ Investigate sensitivity to choice of interpolator and representativeness quality of mapped Jacobians.

Interpolators

Interpolators for data assimilation:

- Nearest neighbour log-linear interpolator (operationally applied at EC for example)
- Proposed alternative: piecewise weighted averaging log-linear interpolator

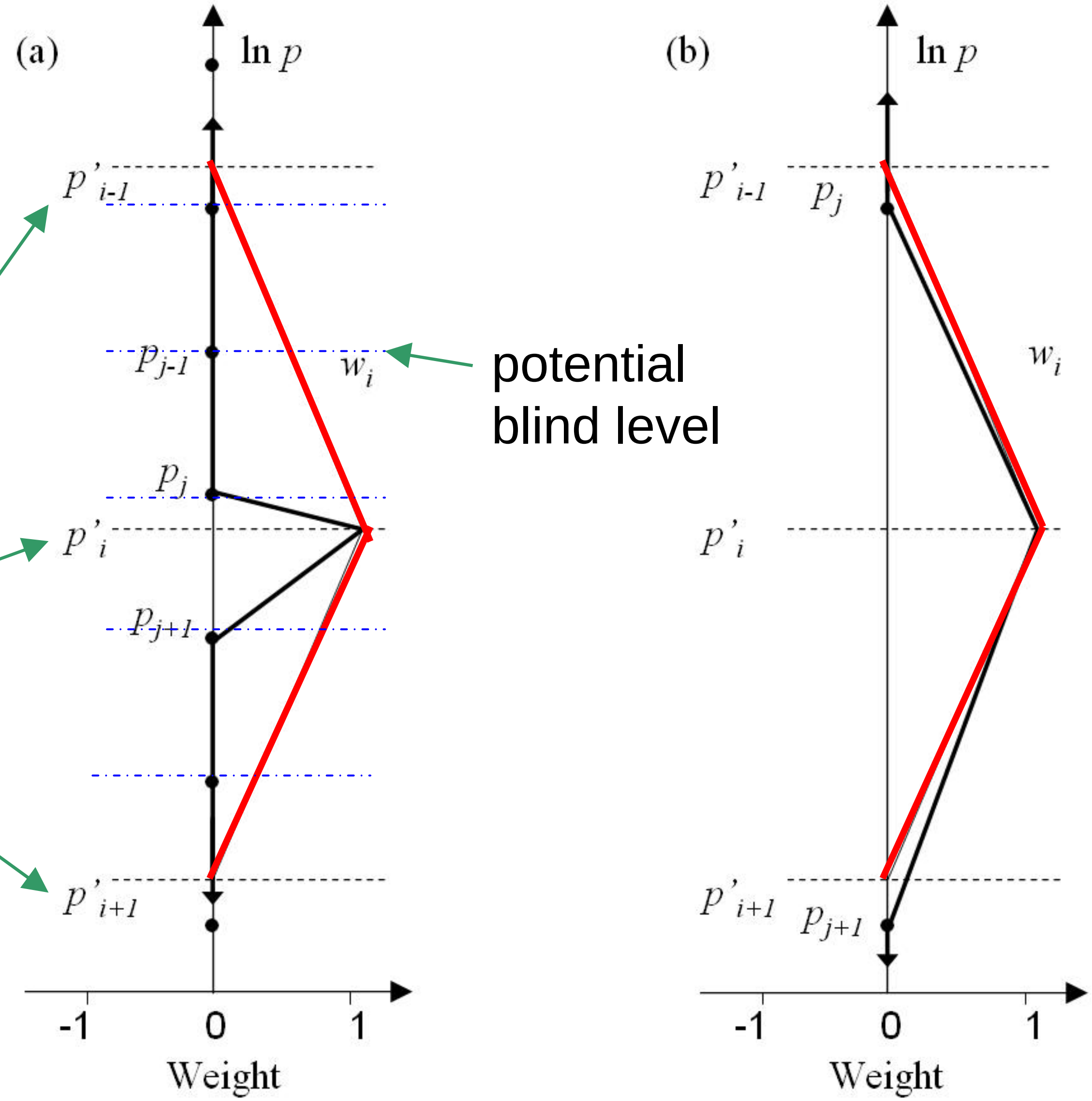
$$x'_i = \frac{\int_{i}^{i+1} \mathrm{w}_i \, \mathrm{x} \cdot d \ln p + \int_{i-1}^{i} \mathrm{w}_i \, \mathrm{x} \cdot d \ln p}{\int_{i}^{i+1} \mathrm{w}_i \cdot d \ln p + \int_{i-1}^{i} \mathrm{w}_i \cdot d \ln p}$$

evaluated using the trapezoidal rule with weights w …

Weighting functions:

Nearest neighbour and piecewise weighted avg. interpolators

RTM levels



Mapping comparisons

Jacobian mappings via adjoint of:

- Nearest neighbour interpolator
- Proposed interpolator

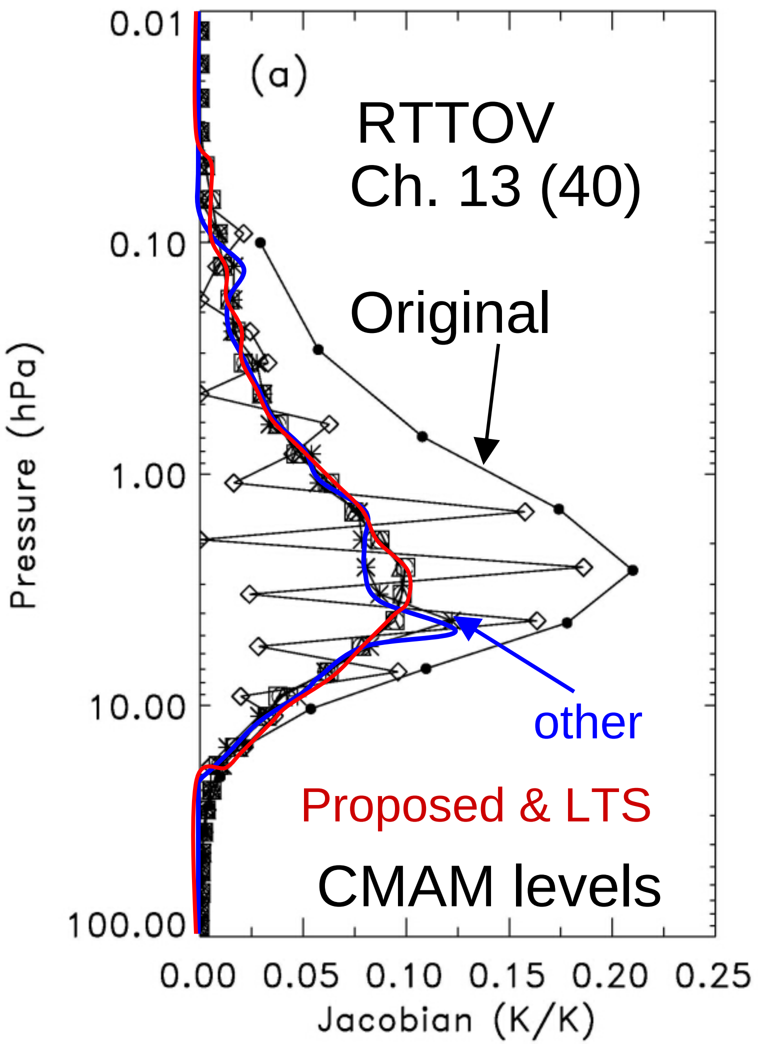
Compared to

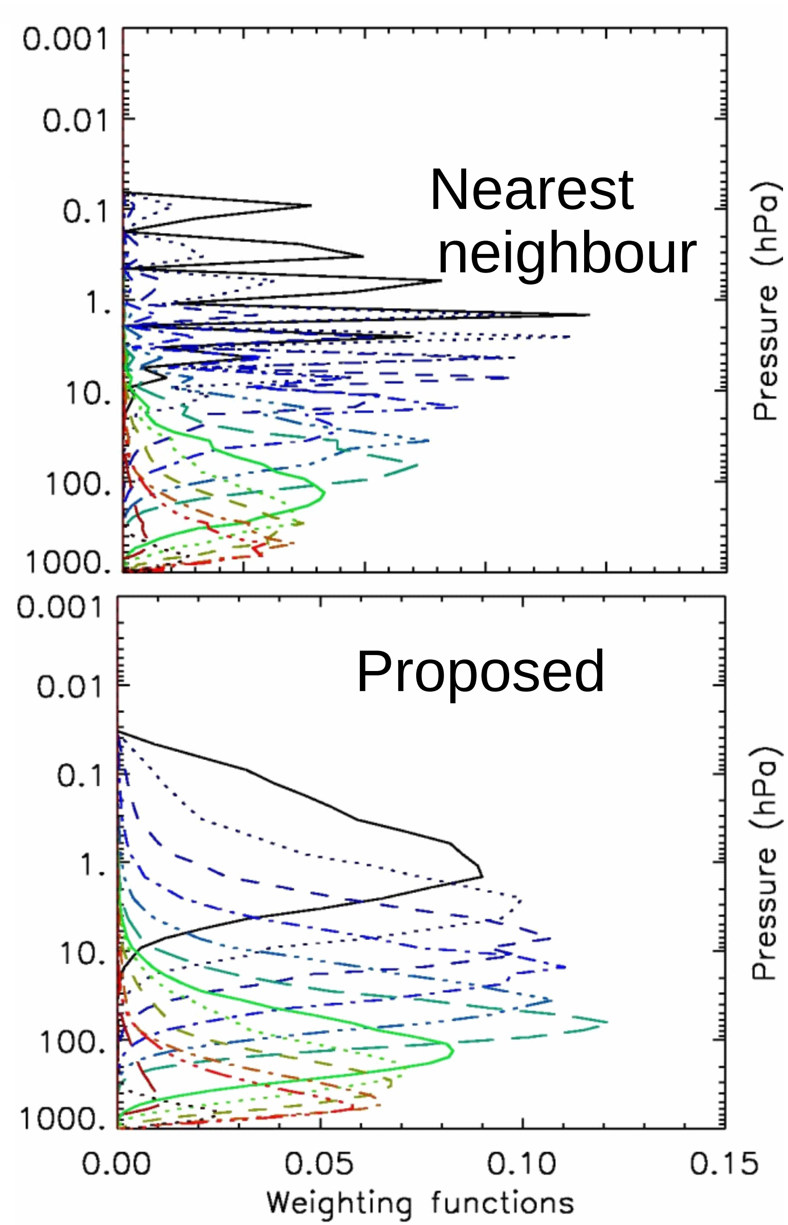
- Layer Thickness Scaling (LTS) interpolation for Jacobian mapping (no forward interpolator and adjoint pairing – not applicable to DA)
- RTM calculations on model levels (D.S. Turner)

using AMSU-A channels up to 14 and GFLBL (D.S. Turner) Jacobian calculations for AIRS (5) and HIRS (5) channels.

N.B.: LTS mapping method was used in Saunders et al. and Garand et al. RTM intercomparisons.

Mapping of AMSU-A Jacobians





Jacobian mappings for HIRS channel 12 for various (M,N)

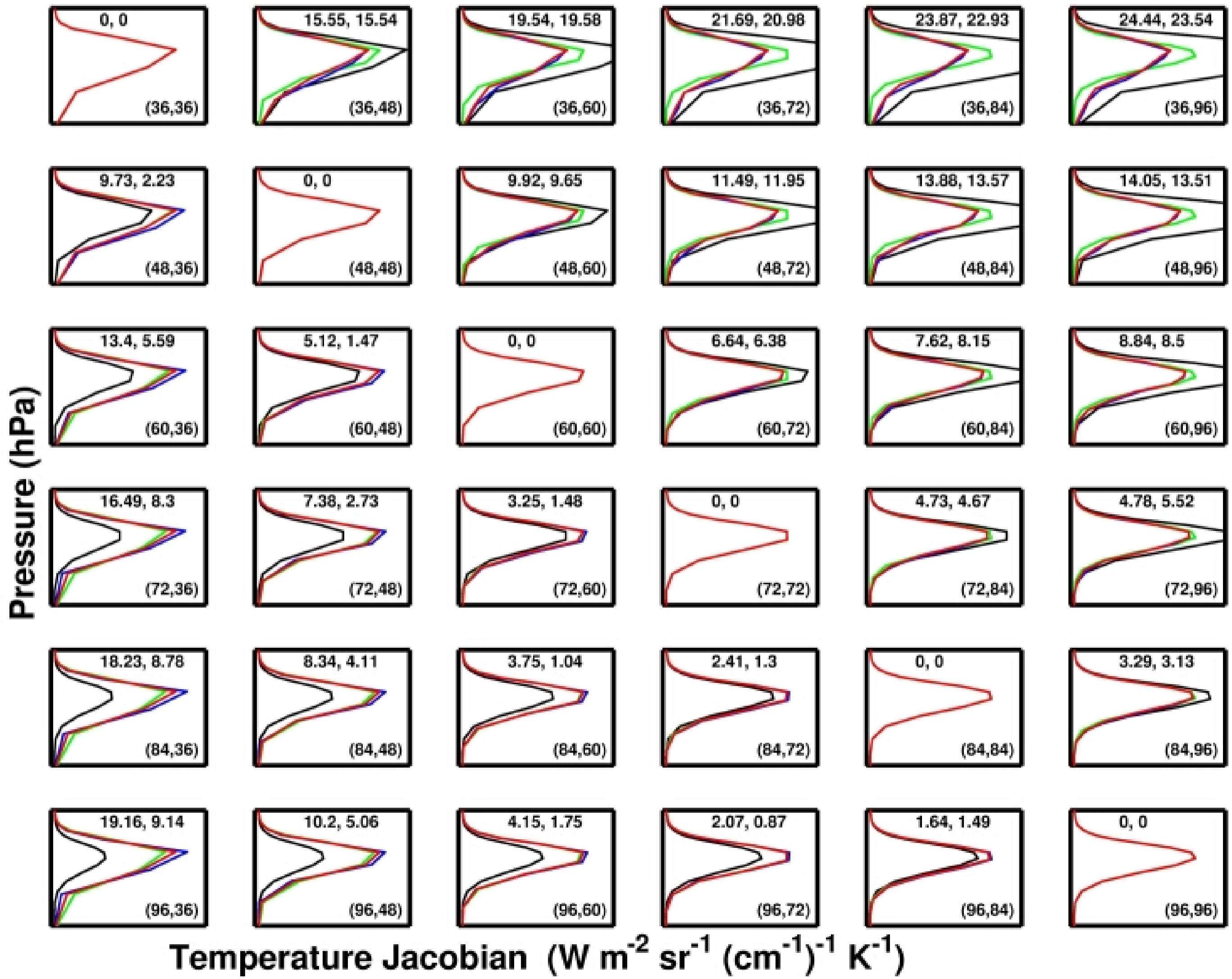
Original from GFLBL

Mapped via
Proposed
LTS

Ref.: GFLBL

Profile relative error measure (%) over AIRS and HIRS channels and various (M,N):

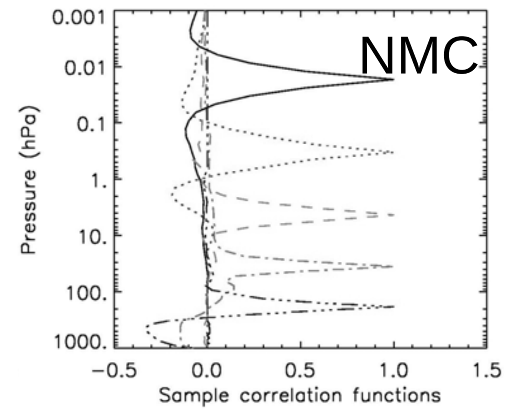
71% with <5%
90% with <15%
for 17 280 cases

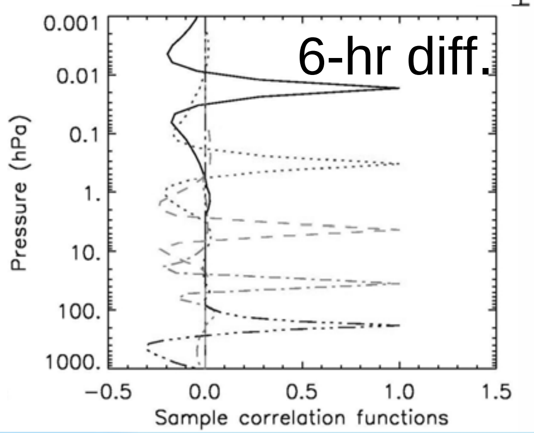


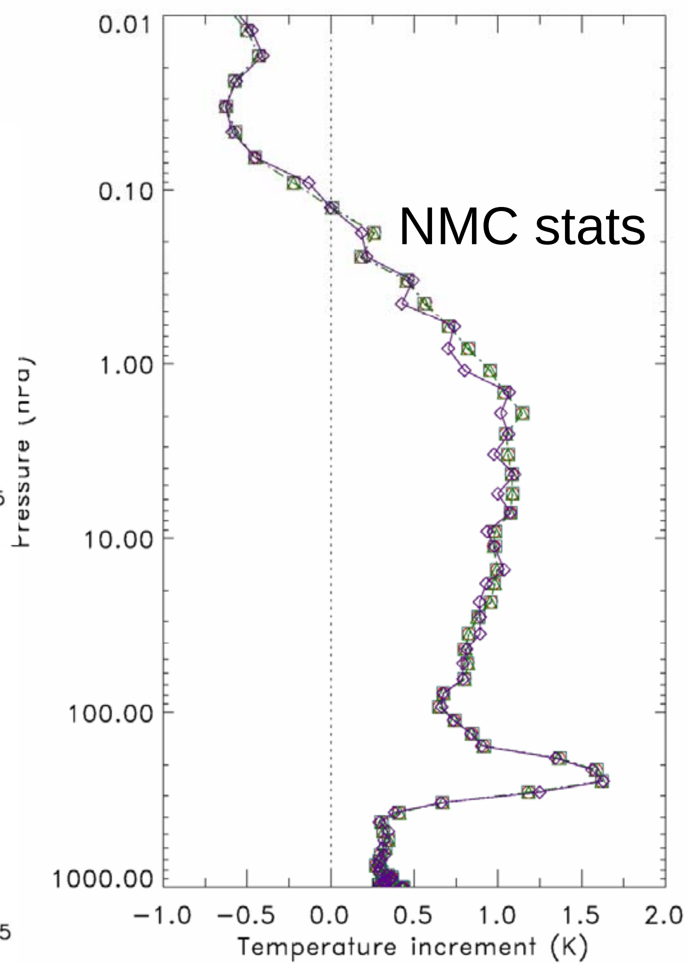
1D assimilation: Impact of vert. correl. & vert. interpolators

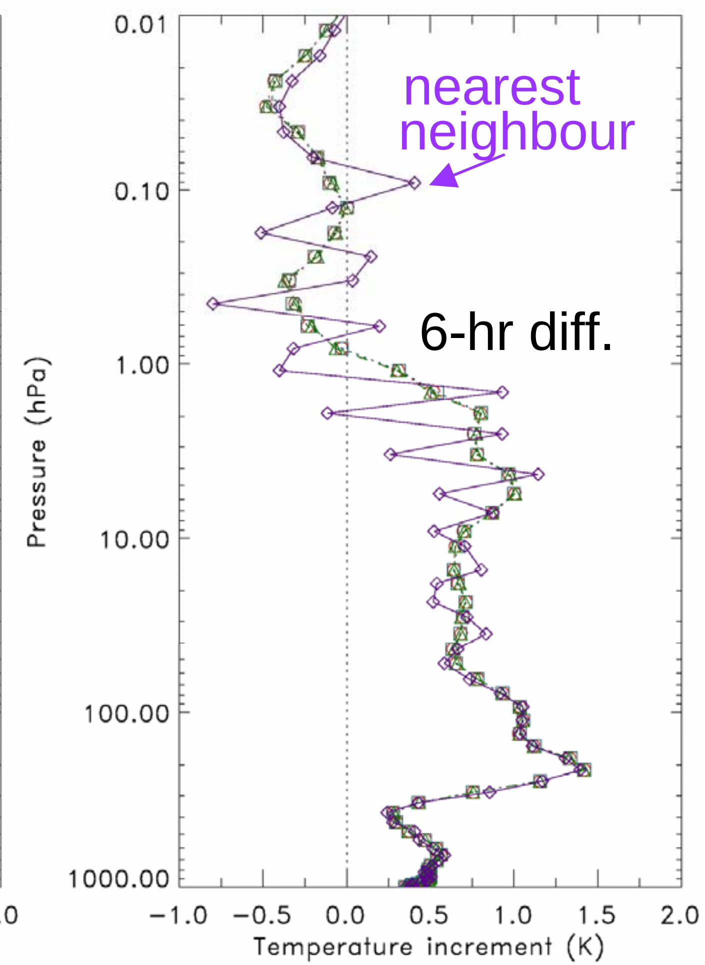
Sample temperature increments

Sample vert. correlation fns

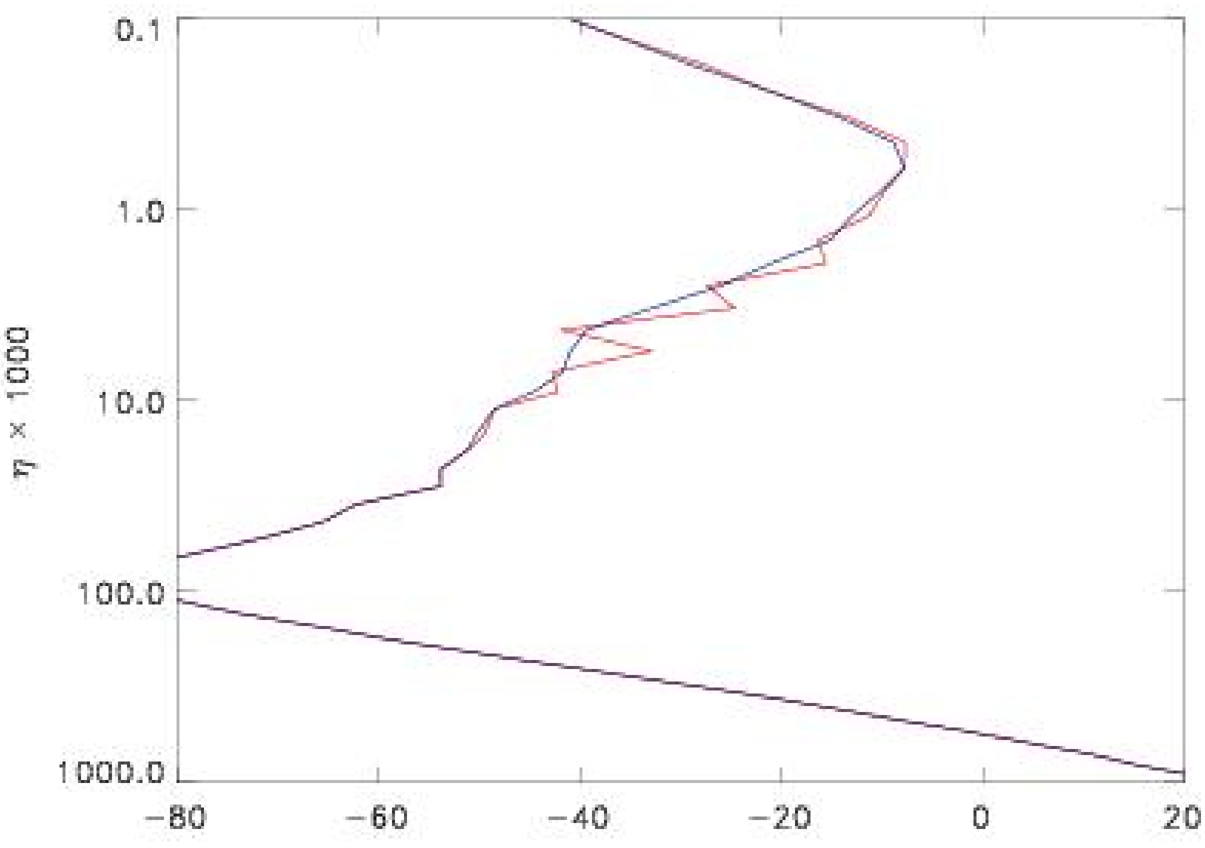








3D-Var assimilation: Diagonal vert. correlation matrices



Temperature (degrees Celcius)

Average analysis profiles over 5 days at the equator

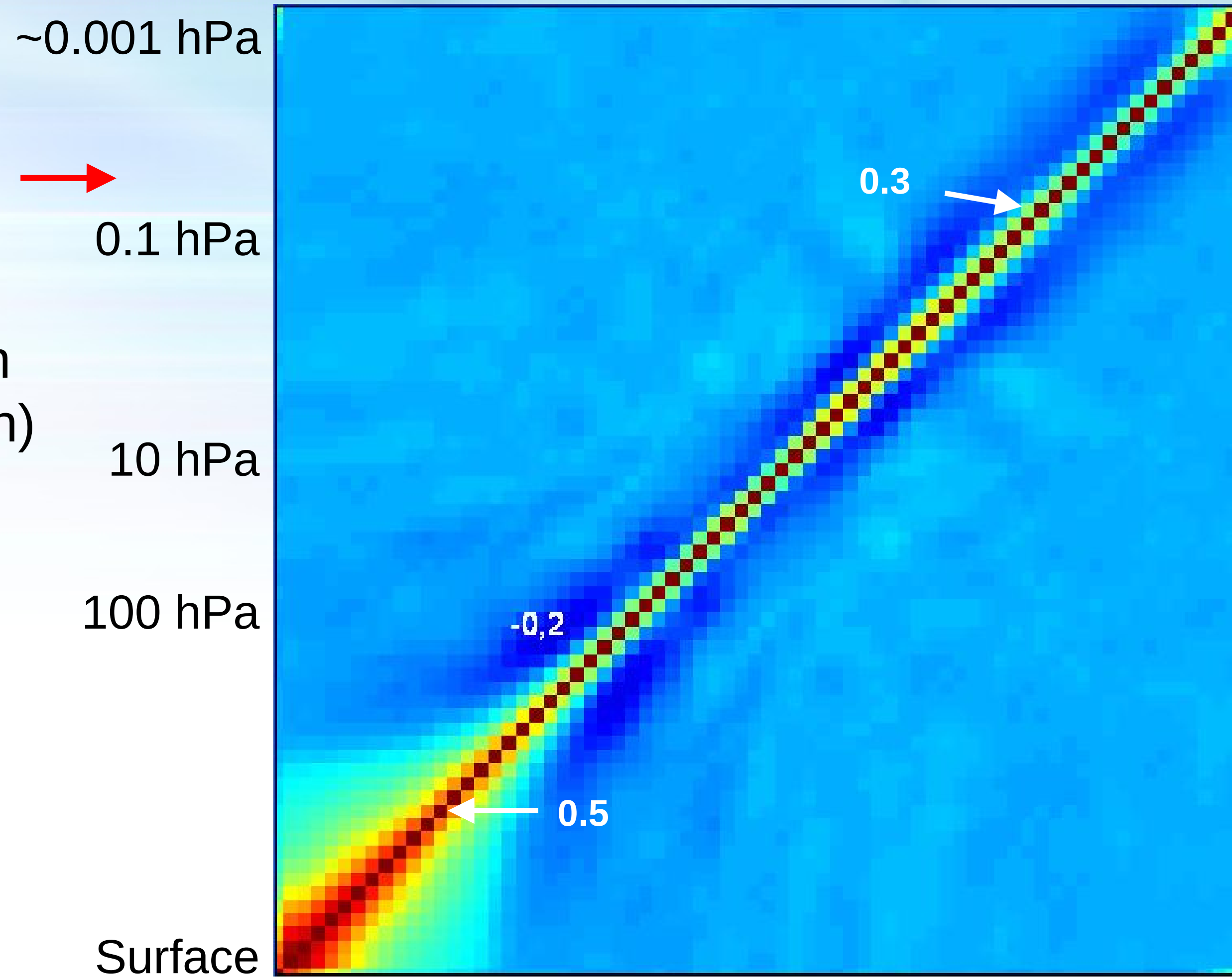
Nearest neighbour

Proposed

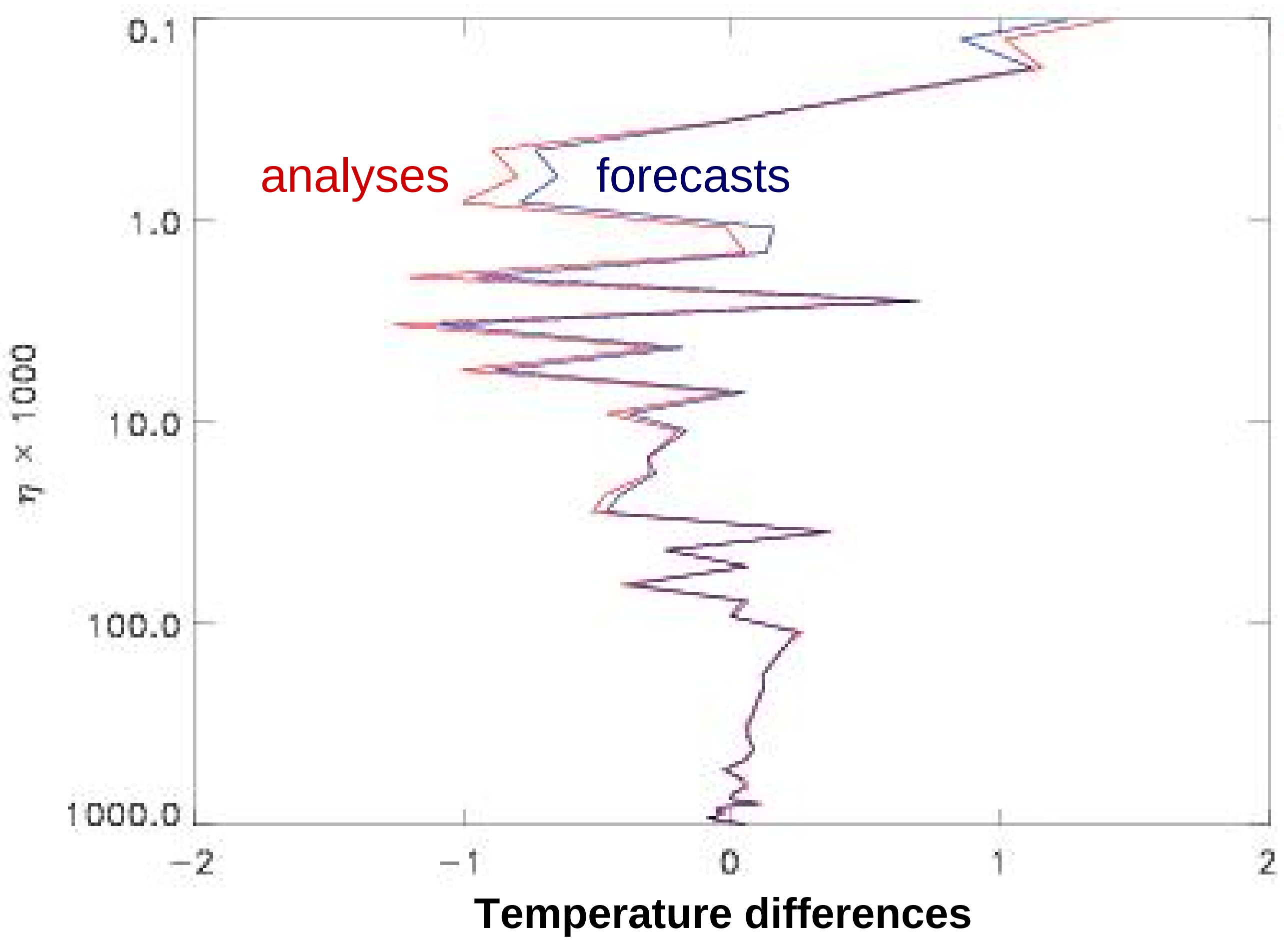
3D-Var assimilation: Impact of vert. correlation & vert. interpolators

CMAM-DA:

vertical correlation matrix from an ensemble perturbation approach (Yulia Nezlin)



3D-Var assimilation: Ensemble perturbation scheme vert. correlation matrices



Average profile differences over 5 days at the equator

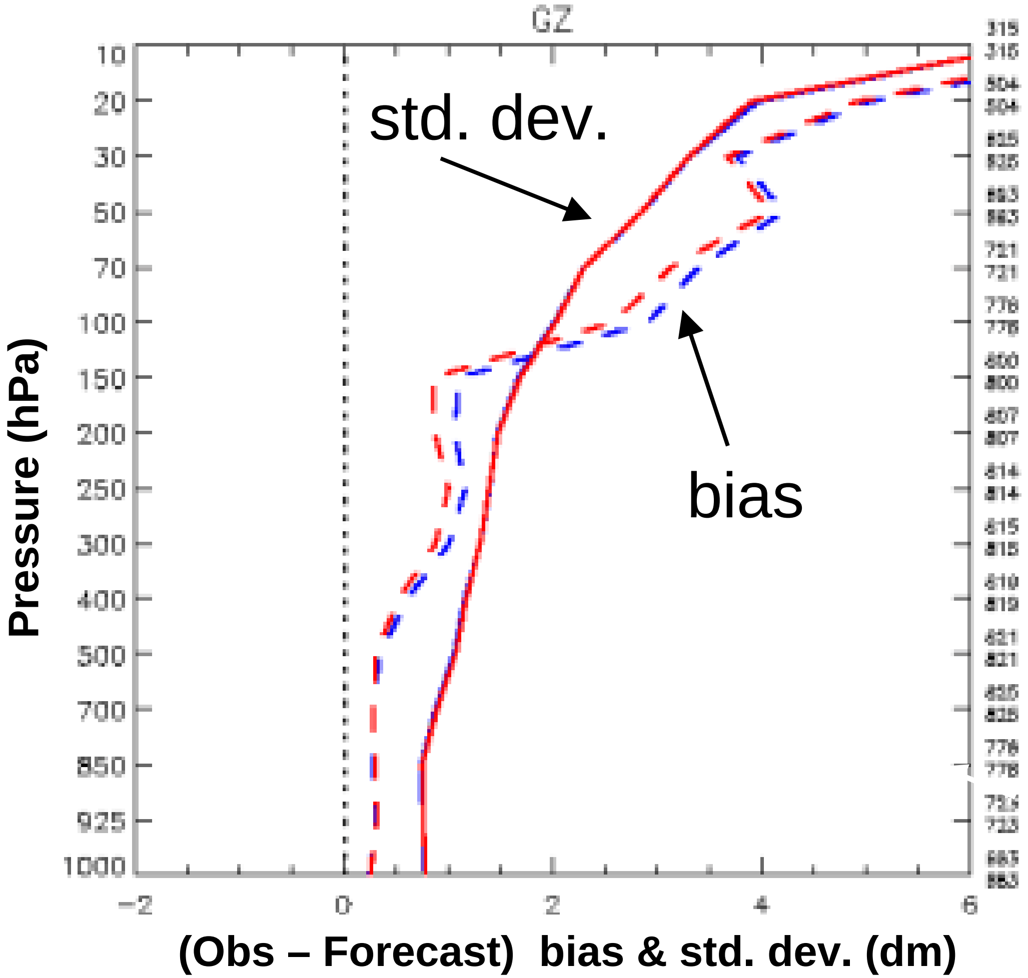
for both analyses and forecasts.

Curves show differences of temperatures obtained from using

- nearest neighbour
- proposed

methods.

3D-Var assimilation: Impact on geopotential height (GEM model and NMC statistics: preliminary results)



For 6-hours forecasts in the tropical region.

Based on 12 days.

Nearest neighbour

Proposed

Summary and comments

- ✓ Proposed vertical interpolator satisfies Jacobian mapping requirements.
- ✓ P.S.: The forward vertical interpolator and its adjoint can account for surface pressure dependency of model coordinate when required.
- ✓ Level of benefit depends on vertical resolutions and width of vertical correlation functions.
- ✓ Stand-alone code to be made available shortly (contact: yves.rochon@ec.gc.ca and louis.garand@ec.gc.ca)
- ✓ Manuscript to QJRMS conditionally accepted.

Thank you!

Extras

LIST OF AIRS and HIRS CHANNELS FOR WHICH SIMULATIONS WERE PERFORMED. HWHM STANDS FOR THE HALF-WIDTH AT HALF-MAXIMUM OF THE JACOBIAN PROFILE

Channel	Frequency (cm^{-1})	Pressure (hPa) at			Related atmospheric variable(s)
		peak	lower HWHM	higher HWHM	
AIRS 305	737.1	750	440	960	temperature
AIRS 453	793.1	900	670	1010	temperature and water vapour
AIRS 1090	1040.1	25	12	80	ozone
AIRS 1766	1544.3	340	260	400	water vapour
AIRS 2197	2500.3	920	670	1010	temperature
HIRS 1	668.9	2	0.3	50	temperature
HIRS 7	749.6	800	490	980	temperature
HIRS 8	898.7	820	620	970	surface temperature and cloud detection
HIRS 9	1028.3	25	1	90	ozone
HIRS 12	1481.0	400	280	550	water vapour

Distribution of goodness of fit measure *m* for four bounded ranges.

$$m = \left\{ \frac{\sum_{i=1}^{N} \left(y_i - y_i^{ref} \right)^2}{\sum_{i=1}^{N} \left(y_i^{ref} \right)^2} \right\}^{1/2} \times 100\,\%$$

17 280 cases