The testing and planned implementation of variational bias correction (VarBC) at the Met Office

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Abstract

A variational bias correction (VarBC) system has been tested at the Met Office. It is planned to use VarBC operationally from mid-March 2016 for the bias correction of satellite sounding instruments in the Met Office's global model. Only the bias of actively assimilated channels will be varied in the initial system. The bias correction applied to channels which are only used for quality control will continue to be generated using the current system, which is based on the method of Harris and Kelly. The predictors used for the majority of satellite sounding instruments will continue to be the 850-300 hPa thickness and 200-50 hPa thickness. The VarBC system is configured so that the biases evolve according to a user-specified bias halving time. Four Legendre Polynomial predictors will be used to analyse small corrections to the static scan bias corrections. SSMIS data suffers from complex biases driven by solar heating and solar intrusions. The bias of SSMIS is modelled using a Fourier series where the phase angle is the position in the orbit measured from the intersection of the orbital plane and the ecliptic.

Extensive testing of VarBC has been conducted, covering winter and summer seasons from 2013, 2014 and 2015, as well as a 7.5 month trial and control. Results have been consistently positive against analysis for geopotential heights, temperature and winds over all seasons tested. Short range forecasts errors of extra-tropical geopotential height as verified against analysis are typically improved by 5-10% (RMSE). Results against sonde geopotential heights were more mixed, showing apparently large degradations in RMS errors at short range, relaxing to improvements at forecast day 3 and beyond. Investigations showed this to be most likely due to the anomalously large role played by biases in the short range, coupled with the intrinsically large uncertainties in sonde geopotential height estimates, most evident at forecast days 1 and 2. The new analyses are typically colder and drier than current analyses with reduced spin-down through the forecast range. Fit to observations are improved, for example the standard deviation of the background fit to ATMS channels is improved by between 2 and 6%.

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1 Introduction

Satellite radiance observations are biased relative to their model background equivalents. The bias may be a combination of instrumental effects, systematic errors in the radiative transfer model, or the bias of the forecast model. For the maximum benefit to forecast accuracy, it is essential that satellite observations are bias corrected before assimilation.

The current bias correction scheme used at the Met Office is here referred to as the *static scheme*. The static scheme is loosely based on the method of Harris and Kelly [Harris and Kelly, 2001], and is briefly outlined in section 2.1. In the static scheme the bias correction is fitted using roughly a month's worth of data. Typically the bias correction is updated at 6-12 month intervals, or as required by significant shifts in the monitored residual biases.

Many other NWP centres, including ECMWF, Meteo-France and NCEP use a variational bias correction scheme (VarBC) [Auligné et al., 2007] where the bias correction is continuously updated as part of the main assimilation. VarBC has now been implemented and tested at the Met Office. The scheme developed at the Met Office follows quite closely the formulation described by Auligné, and is outlined in section 2.2. Recent enhancements to VarBC and planned future development are summarised in sections 2.3 and 2.4.

VarBC has been extensively tested at the Met Office, including a 7.5 month trial and control. The results from VarBC have been consistently positive and the main features are presented in section 3. As stated in section 4 the source of the positive impact is still under investigation. Conclusions and areas for future work are summarised in section 5.

2 Bias correction at the Met Office

2.1 Overview of the static bias correction scheme

The satellite radiance bias correction scheme currently used at the Met Office, here referred to as the *static* bias correction scheme, is based on the method of Harris and Kelly [Harris and Kelly, 2001].

The bias is modelled as a constant, a scan bias correction, and a set of coefficients and predictors:

$$y_k^o := y_k^o - \left(c_k + s_k + \sum_{i=1}^{I_k} \beta_i^b p_{k,i} \right) \tag{1}$$

following the notation in appendix A. The scan bias is adjusted so that the mean scan bias across all scan positions is zero (or that the mean scan bias of specific scan positions is zero); this is why a separate constant offset is required. The air-mass predictors used are the model 850-300 hPa thickness and 200-50 hPa thickness.

New bias corrections are typically generated using between 10 days' and one month's worth of data, although longer periods have been used. The bias correction updated infrequently, typically every 6–12 months, or if there has been a significant change in the bias as flagged by satellite radiance monitoring.

2.2 Main features of VarBC

The variational bias correction scheme implemented at the Met Office follows quite closely the incremental formulation described by Auligné [Auligné et al., 2007].

The bias model used in VarBC is very similar to that of the static scheme:

$$y_k^o := y_k^o - \left(s_k + \sum_{i=1}^{I_k} \beta_i^b p_{k,i} \right) \tag{2}$$

The main difference is that the first predictor is set to be 1 so that the first coefficient takes on the role of the constant bias offset. The ability to apply a scan-position by scan-position offset, s_k, has been retained, although as implemented there is currently no mechanism to generate or update this scan bias correction in VarBC.

The bias correction is applied to observations in the 1D-Var pre-processing and quality control step. An increment to the bias coefficients is then analysed as part of the data assimilation stage. The data assimilation stage minimises a penalty function, and the observation part of the penalty function includes an increment to the bias correction:

$$J_o = \frac{1}{2}\sum_k \left(\left(y_k + \sum_{i=1}^{I_k} \beta_i' p_{k,i} - y_k^o \right) R_k^{-1} \left(y_k + \sum_{j=1}^{I_k} \beta_j' p_{k,j} - y_k^o \right) \right) \tag{3}$$

The coefficients are not included in the control vector directly but are derived from the control vector by a transformation designed to improve the conditioning of the problem, following the approach of Dee [Dee, 2004]:

$$\beta' = \mathbf{U}_\beta \mathbf{v}^\beta \tag{4}$$

To prevent the coefficients being made too specific to the current assimilation cycle a background term limits how far the coefficients can change in any given cycle:

$$J_\beta = \frac{1}{2}\sum_{i=1}^{I_k} \beta_i'^T V_{(\beta_i)}^{-1} \beta_i' \tag{5}$$

At most centres the size of the background error term ($V_{(\beta_i)}$) is controlled by constant called N_{bgerr}. When N_{bgerr} equals the number of observations that use the predictor then the weight of the prior and the weight of the observations are equal in determining the analysed β_i'. At the Met Office the N_{bgerr} is set as:

$$N_{bgerr} = \mathtt{MAX}(m_{avg}, M_{min}) \left(\frac{1}{2^{\frac{1}{n}} - 1} \right) \tag{6}$$

where m_{avg} is the expected number of observations per data assimilation cycle for the channel in question, M_{min} is a minimum number of observations, and n is the bias halving time chosen by the user. This approach to setting N_{bgerr} is discussed further in appendix B.

An example of VarBC working to correct the bias of a channel on FY3C is shown in figure 1.

2.3 Recent enhancements to VarBC

The current implementation of VarBC at the Met Office includes several enhancements, in both the predictor model and the adaption rate of bias coefficients, relative to the scheme implemented at most other NWP centres.:

Harmonised adaption rates: In the initial implementation, the adaption rate of the bias coefficients could vary from channel to channel for a given observation type. Channels which were assimilated in small numbers would take longer to adapt than bias corrections for channels present in greater numbers. Slow adaption of the bias correction is not necessarily problematic provided the system is able to keep up with seasonal, or other, variations in bias, however from a practical perspective it is helpful if the system converges rapidly. Too rapid adaption of the bias correction can be problematic where not enough data has been collected to properly sample the full range of bias variations around the globe, or the full range of diurnal conditions. The use of a bias halving time ensures that all channels are able to strike a balance between

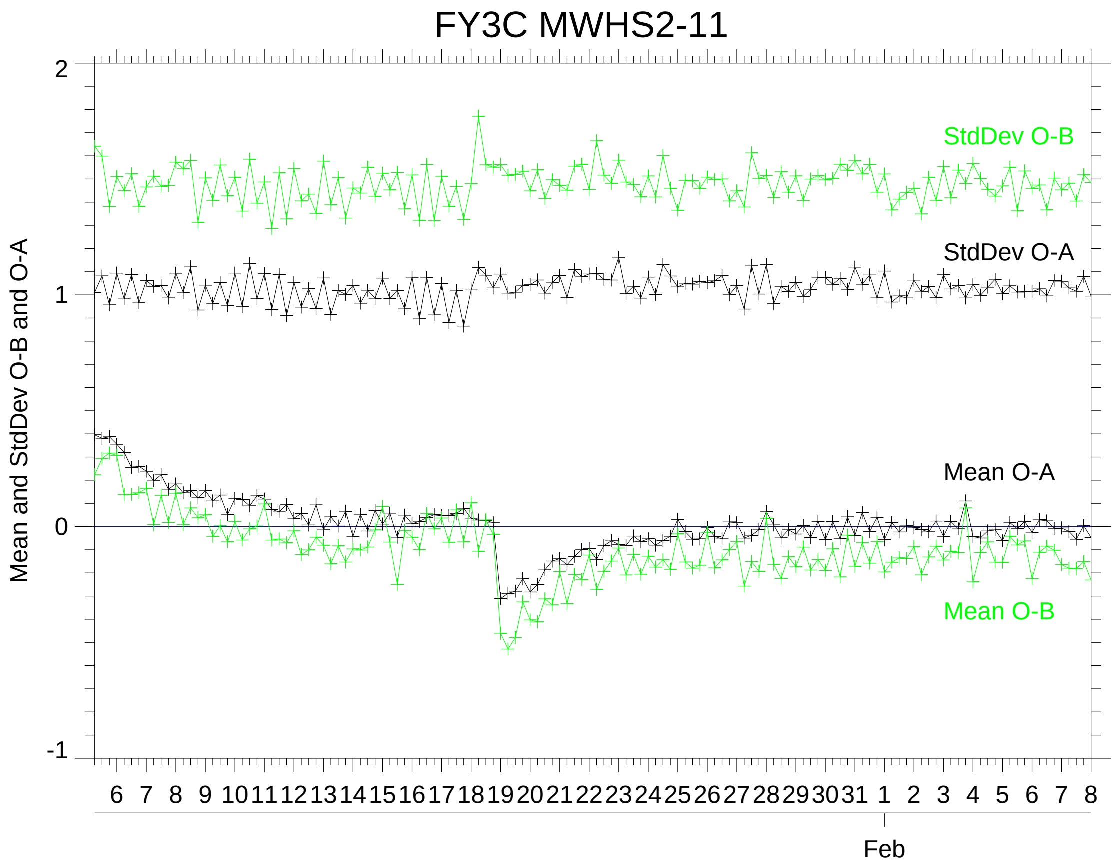


Figure 1: The mean and standard deviation of observed minus background (O-B) and observed minus analysis (O-A) for a channel on FY3C for each data assimilation cycle taken from the start of an assimilation trial. The initial bias is not perfect and VarBC improves the correction with a bias halving time of around 2 days. A planned change was made to the way the data was processed at CMA on 19 January, resulting in a change in the instrument bias. VarBC again adjusts the bias correction with a bias halving time of around 2 days.

proper sampling of the globe and rapid convergence.

Legendre Polynomial scan bias correction: The VarBC system retains the ability to apply a static offset to each scan position. There is also the option to apply a set of (orthogonal) Legendre Polynomial scan bias predictors to correct any residual scan bias, see figure 2.

Fourier series orbital bias predictors: As documented by Booton [Booton, 2014] and Bell [Bell, 2008] the Special Sensor Microwave Imager/Sounder exhibits complex biases due to several sub-optimalities in the calibration of the instrument. These are primarily manifested as orbital biases. A scheme has been developed to correct for these biases based on a Fourier series expansion in the orbital angle.

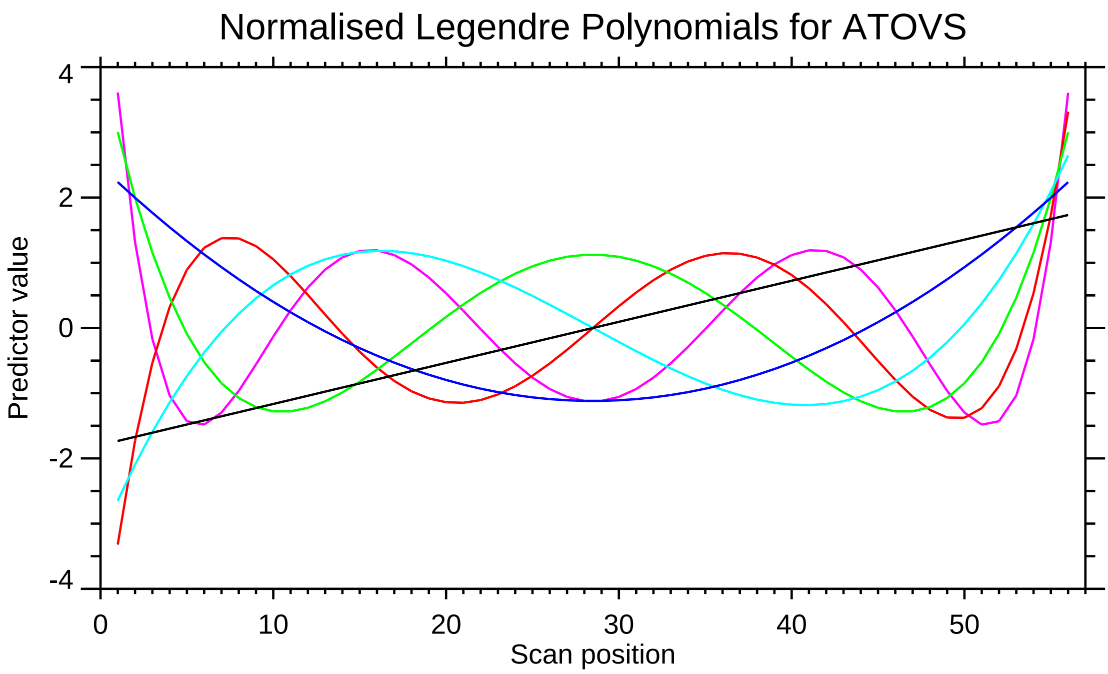


Figure 2: The first 6 Legendre Polynomials in scan position for ATOVS, normalised to a standard deviation of 1.

2.4 Future development of VarBC

There are a number of ways in which the implementation of VarBC at the Met Office may be improved in the future:

Passive channels: In the VarBC scheme at the Met Office there is not presently any mechanism to generate a bias correction for channels that are used for quality control in the 1D-Var but are not assimilated. These channels are currently still bias corrected using the static scheme.

Observation selection: It is planned to introduce a mechanism to select which observations affect the bias correction based on surface and cloud type.

The scan bias: The VarBC scheme at the Met Office retains the option to apply an offset to each scan position, but there is currently no automatic mechanism for generating or updating these values.

Regional models: It is planned to test running VarBC actively in a regional model and compare this to a regular update of the bias correction from the global model.

Non-satellite data: Both schemes currently only apply to satellite radiance observations. There are currently no plans to extend VarBC to other observation types.

3 Performance of the VarBC trials

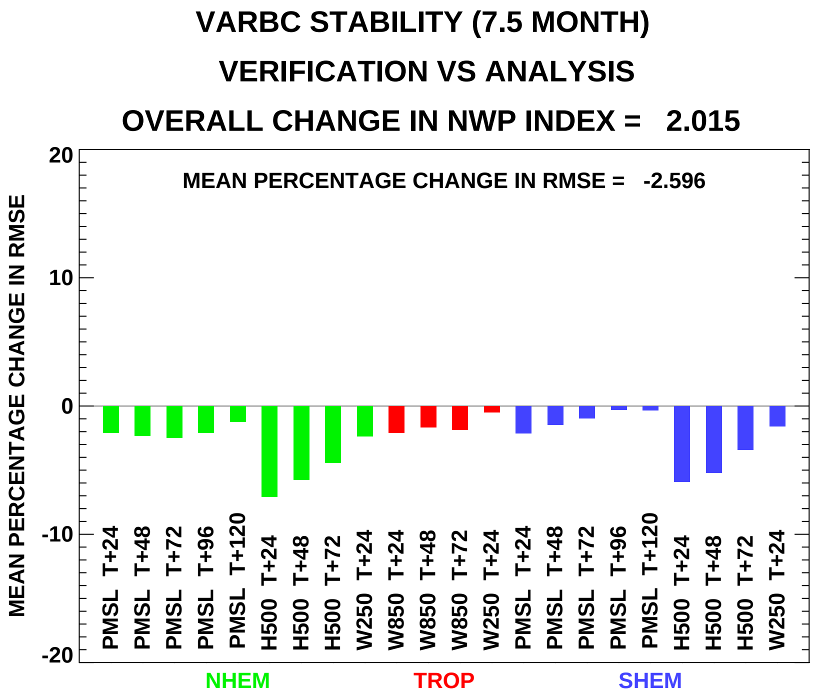
VarBC has been tested, in various configurations, for summer and winter seasons of 2013, 2014 and 2015, and for a 7.5 month (230 day) period from 2 Dec 2014–20 July 2015.

The verification against analysis and against observations for the 7.5 month VarBC stability run is shown in figure 3. The PMSL and 500 hPa heights in the extra-tropics, 250 hPa and 850 hPa winds in the tropics are improved at all forecast ranges examined when verified against analysis. There are 7.1% and 5.9% improvements in the RMSE of 500 hPa heights at T+24 for the Northern and Southern Hemispheres respectively. The verification against observations shows some apparently large degradations to the fit of 500 hPa heights at T+24, with the effect declining with forecast lead time, and, in the case of the Northern Hemisphere, even reversing sign. As will be shown, this degradation is actually due to a shift in the bias of the forecast 500 hPa heights, as verified against sondes. These results are typical of the VarBC trials. Figure 4 shows the time series of the NWP index, verified against analysis, during the 7.5 month run. It shows fairly consistent good performance throughout, as evidenced by the stable running mean.

The VarBC trials were found to produce colder, drier analyses, and less rainfall in the hours following a data assimilation cycle. Figure 5 shows the difference in zonal temperature at T+6 between a winter VarBC trial (2 December 2014 - 12 January 2015) and the control. The VarBC experiment is generally cooler, but especially at 850 hPa. Figure 5 also shows a map of the mean difference in temperature at 850 hPa between trial and control. The change in temperature is primarily over the oceans and especially in marine stratocumulus regions. Figure 6 shows the difference in zonal relative humidity between trial and control. The VarBC trial is drier at the lowest levels, especially in the sub-tropics of the summer hemisphere.

The lower temperatures in VarBC, shown in figure 5, naturally lead to lower values of 500 hPa height. Figure 7 shows the verification of 500 hPa height against sondes, versus forecast range, for the 7.5 month trial. It can be seen that the VarBC heights are typically a couple of metres lower than for the control. This increase in bias is the reason for the apparently negative verification of 500 hPa height against sondes, but is thought to be within the uncertainty of sonde measurements. The RMS fit of forecast 500 hPa heights is worse at short range due to the increase in bias, however with increasing forecast range the bias becomes a less significant component of the RMS, and in the case of the Northern Hemisphere the RMS error is lower in the VarBC experiment than in the control at long range.

The verification of 500 hPa heights against sondes was investigated further. Figure 8 shows the verification of H500 for ECMWF and the Met Office operational models, relative to both analyses and radiosondes, during the period 2000 - 2015. Figure 8 also shows that at T+24 forecasts now have errors below the noise floor of the radiosondes. The radiosondes are unable to differentiate between the performance of ECMWF and the Met Office models since 2005. In this situation, the radiosondes have become relatively insensitive to changes in forecast performance. At the same



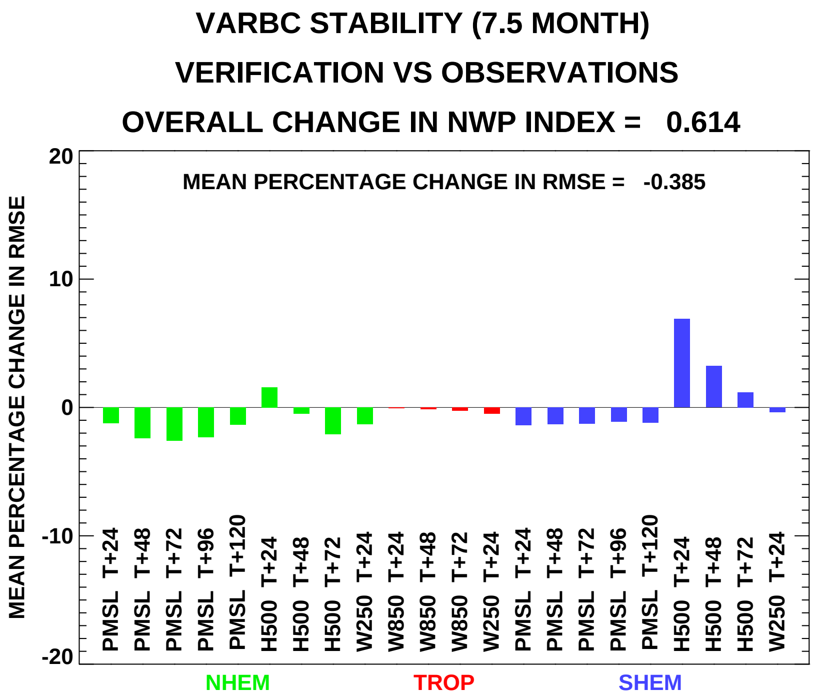


Figure 3: The verification against analysis and verification against observations for the long 7.5 month (230 day) VarBC stability run. The verification against analysis is strong for all variables, especially the 500 hPa heights where the RMS is 7.1% lower in the NH and 5.9% lower in the SH at T+24. In the verification against observations the 500 hPa heights shows apparently large degradations. It will be shown that this is due a change in bias, which is thought to be within the uncertainty of sonde measurements.

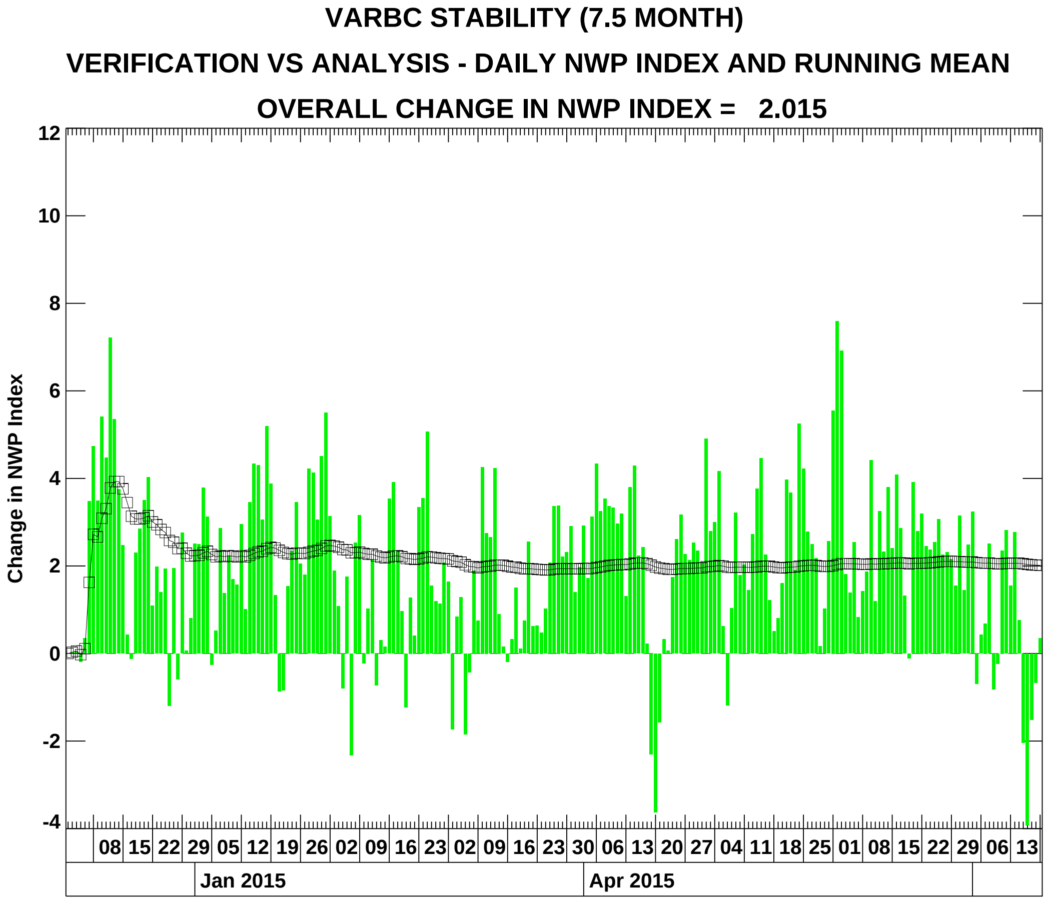
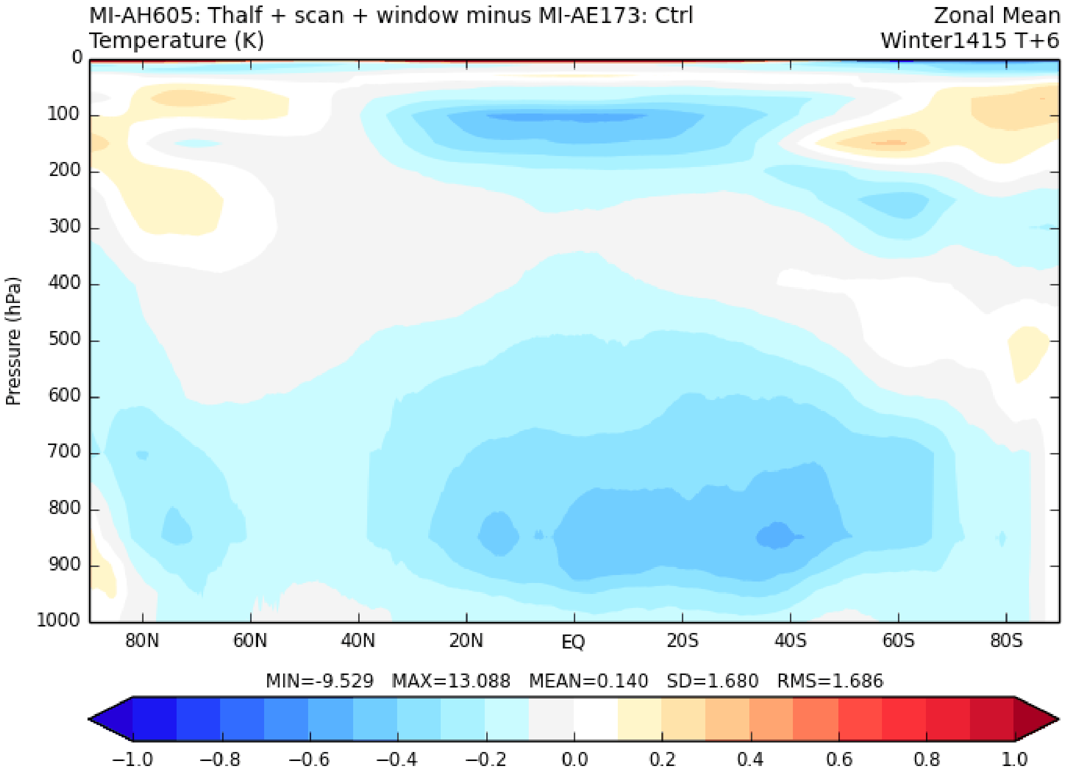


Figure 4: Time series of the NWP index, verified against analysis, for the long 7.5 month VarBC stability trial. The trial shows consistent benefit across the trial, as evidenced by the stable running mean.

time the changes in bias relative to radiosondes have become a more significant driver of changes in RMSE. With T+24 errors at around 5-10 metres, bias changes of 2-3 metres (well within the uncertainty limits of radiosonde measurements) dominate the RMSE changes. This is believed to be the reason for the anomalous performance relative to observations.

Figure 9 shows the verification of the temperature at 850 hPa against analysis for the 7.5 month trial and control. The difference between forecast and analysis tends to become more negative for both trial and control with increasing forecast lead time, however the effect is larger in the control. The VarBC trial is analysing colder temperatures, and therefore has less far to spin down to the preferred model climate. The reduction in spin-down naturally leads to better verification against analysis.

Figure 10 shows the change in convective and large-scale rainfall. Perhaps not surprisingly for a cooler and drier forecast there is typically less rain in the ITCZ and at mid-latitudes at T+24. Verification of the 7.5 month VarBC stability run shows improvements in the UK index scores. The UK index is +0.12 for the Northern Hemisphere, +0.02 for the tropics, and +0.11 for the Southern Hemisphere. The UK index for the British Isles is +0.33 and for the UK index stations +0.31. In all these UK index score results the most striking factor is an improvement in the 6 hour precipitation



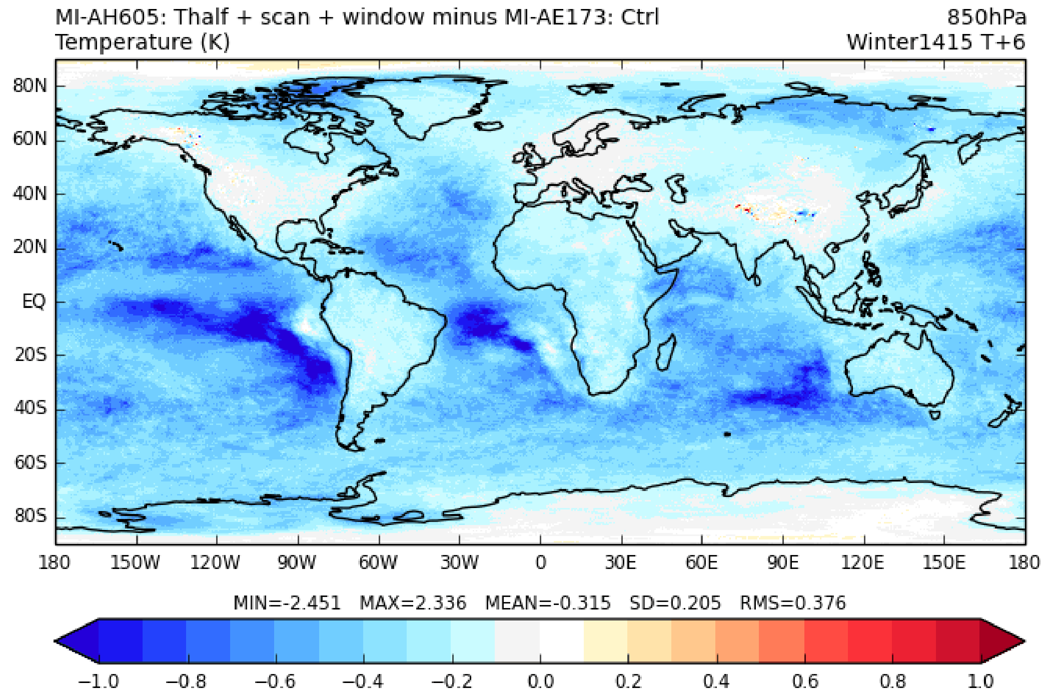


Figure 5: The upper figure shows the zonal mean of the difference in temperature between a winter VarBC experiment and the control. The VarBC experiment is generally cooler, but especially at 850 hPa. The lower figure shows a map of the mean difference in temperature at 850 hPa. The change in temperature is primarily over the oceans and especially in marine stratocumulus regions.

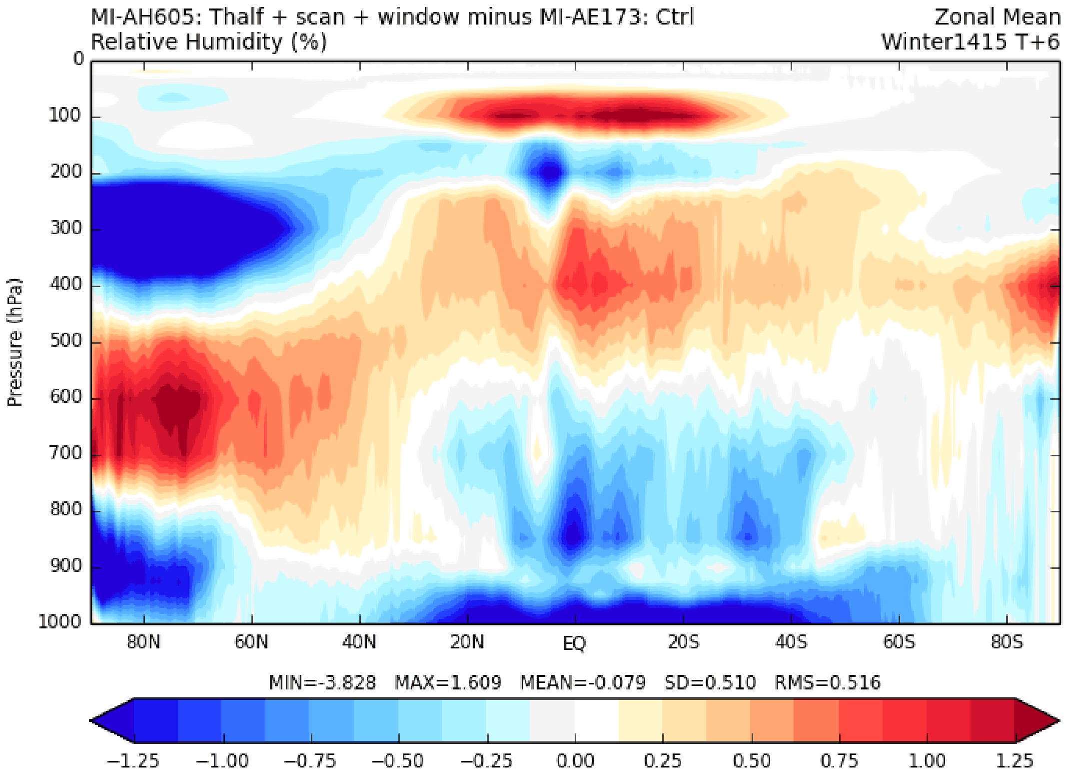


Figure 6: Zonal mean of the change in relative humidity for a winter VarBC experiment minus the control. At the lowest levels the VarBC experiment is predominantly drier.

accumulation equitable threat score.

The bias of AIRS window channels were stable to within 50 mK across the VarBC stability run, however it was discovered that the applied bias drifted by 0.5 K over the 7.5 months (see figure 11). The change in applied bias is offset by changes to the fitted skin temperature, and somewhat to the fitted cloud parameters, in the 1D-Var. These fitted parameters are used in data assimilation for forward modelling but do not feed into the analysis. The static scheme avoided this type of drift by using the first-guess skin temperature in the bias calculation rather than the 1D-Var fitted value. With VarBC the 1D-Var analysed skin temperature is only loosely tied to the first guess skin-temperature through the background error. A trial where the bias corrections of 8–9 window channels on AIRS, IASI and CrIS were held fixed was run for a summer and winter period. These trials gave similar results to before and so it is planned to use this configuration in the operational VarBC system.

Figures 12 and 13 show the mean O-B and O-A for ATMS and MetOp-B IASI for the long stability run trial and control. Note that the mean O-A is essentially zero for all bias corrected ATMS channels, whereas the O-B is positive. VarBC bias corrects to the analysis. The forecast tends to run colder, which leads to a positive O-B at the next assimilation cycle. For IASI it is a similar picture for the long-wavelength CO_2 sounding channels, however the O-A for the window channels are off by around 20 mK.

Figures 14 and 15 show the percentage change in the standard deviation of O-B and O-A for

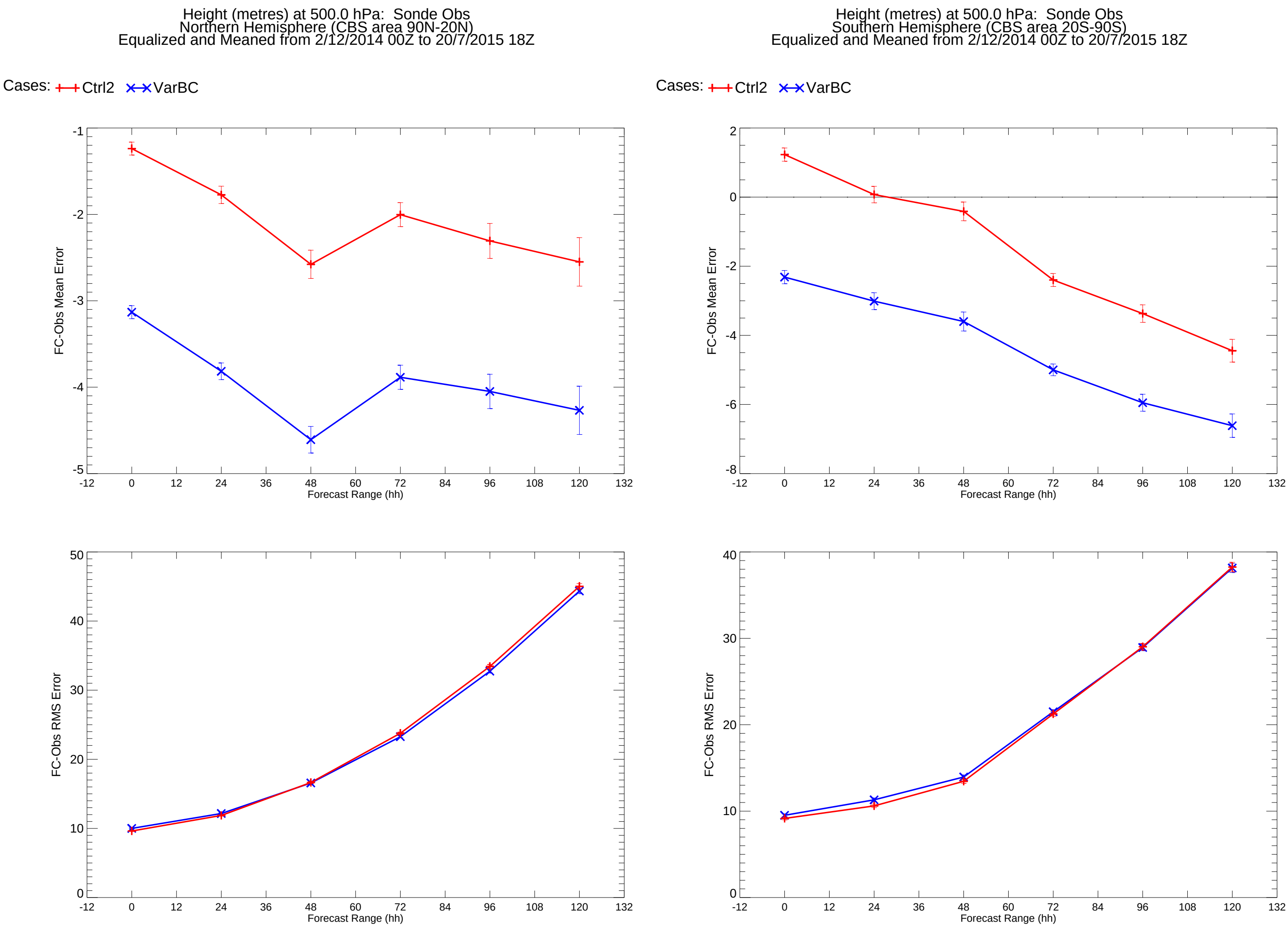


Figure 7: The verification of 500 hPa heights for the 7.5 month VarBC trial and control versus sondes for Northern and Southern Hemisphere regions. The trial is more biased with respect to sondes, but is thought to be within the systematic uncertainty of sonde measurements. The RMS is slightly larger at short range due to the increased bias, but note that at longer lead times in the Northern Hemisphere the RMS is lower in the VarBC trial, despite the larger bias.

ATMS and IASI in the long stability trial. The bias corrected ATMS channels show improvements of 1.5–6% in standard deviation. The long-wavelength CO_2 sounding IASI channels appear well behaved, but the window channels show increases in standard deviation in the range 3–14%. There is an increase in standard deviation of the window channels for all advanced IR sounders. Interestingly this increase is still present when the bias of 8–9 window channels is fixed and it even persists when using static biases derived from the VarBC forecast backgrounds. The behaviour of the advanced IR window channel needs to be investigated further but given the otherwise excellent performance of VarBC, it should not prevent implementation.

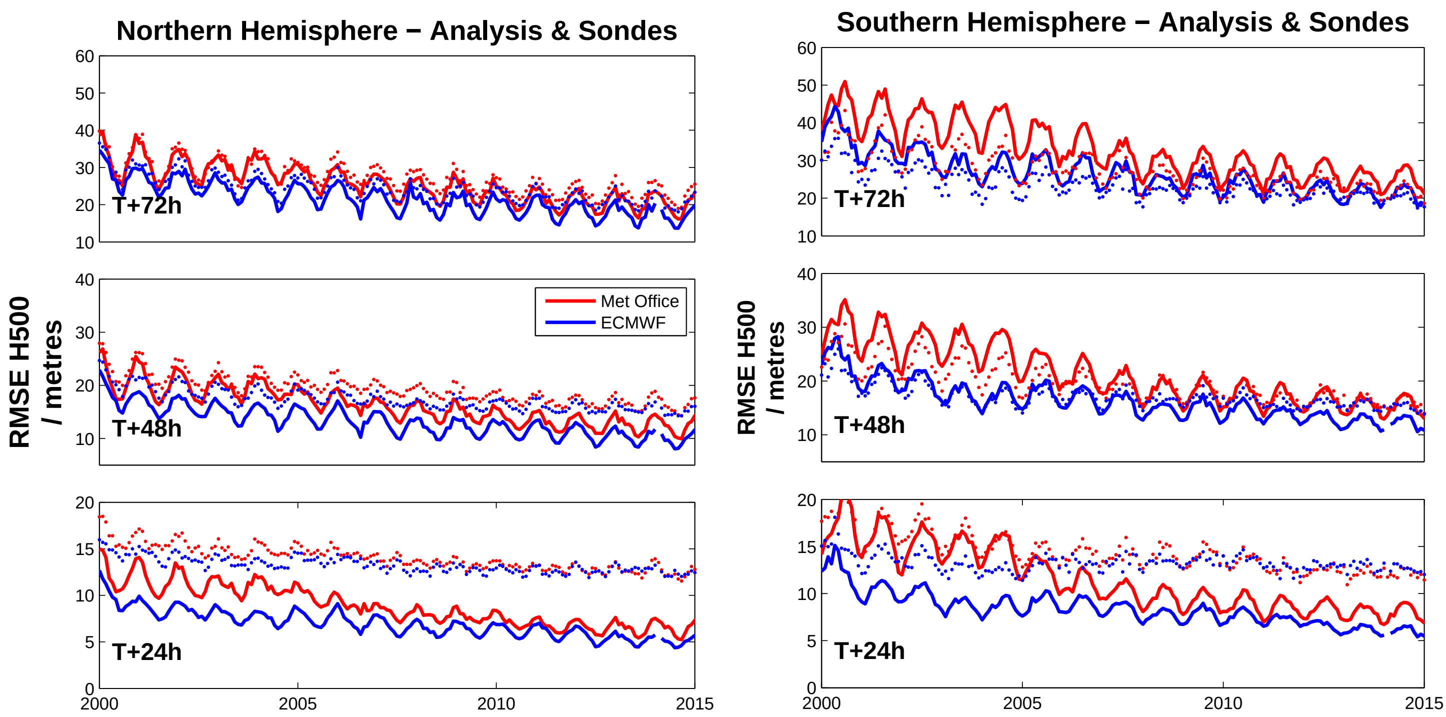


Figure 8: The verification of 500 hPa heights for the Met Office and ECMWF. Northern Hemisphere verification is shown on the left and Southern Hemisphere on the right. The verification against analysis is shown by solid lines and verification against radiosonde 500 hPa heights by dotted lines. The forecast ranges shown are for T+72, T+48 and T+24.

4 The source of the impact of VarBC

The source of the impact of VarBC is still under investigation. VarBC and the static scheme have a similar form for the bias correction and it was not anticipated that VarBC would have such a large impact. At the time of writing the leading hypothesis is that the benefit comes from using exactly the same forward model for bias correction as is used for data assimilation. There are some subtle differences in the forward modelling used in the 1D-Var quality control stage, which drives the static bias correction scheme, and the forward modelling in the data assimilation system which is used in VarBC.

A difference between VarBC and the static scheme is that VarBC bias corrects to the analysis whereas the static scheme corrects to the forecast background. An experiment was run where the bias correction was updated in a separate minimisation where the forecast background was held static by making the background error very small for atmospheric variables. The data assimilation minimisation was run using bias corrected observations and the normal background error, but without actively updating the bias correction in VarBC. This trial gave very similar benefits to the standard VarBC run, indicating that bias correcting to the analysis is not the source of the impact of VarBC.

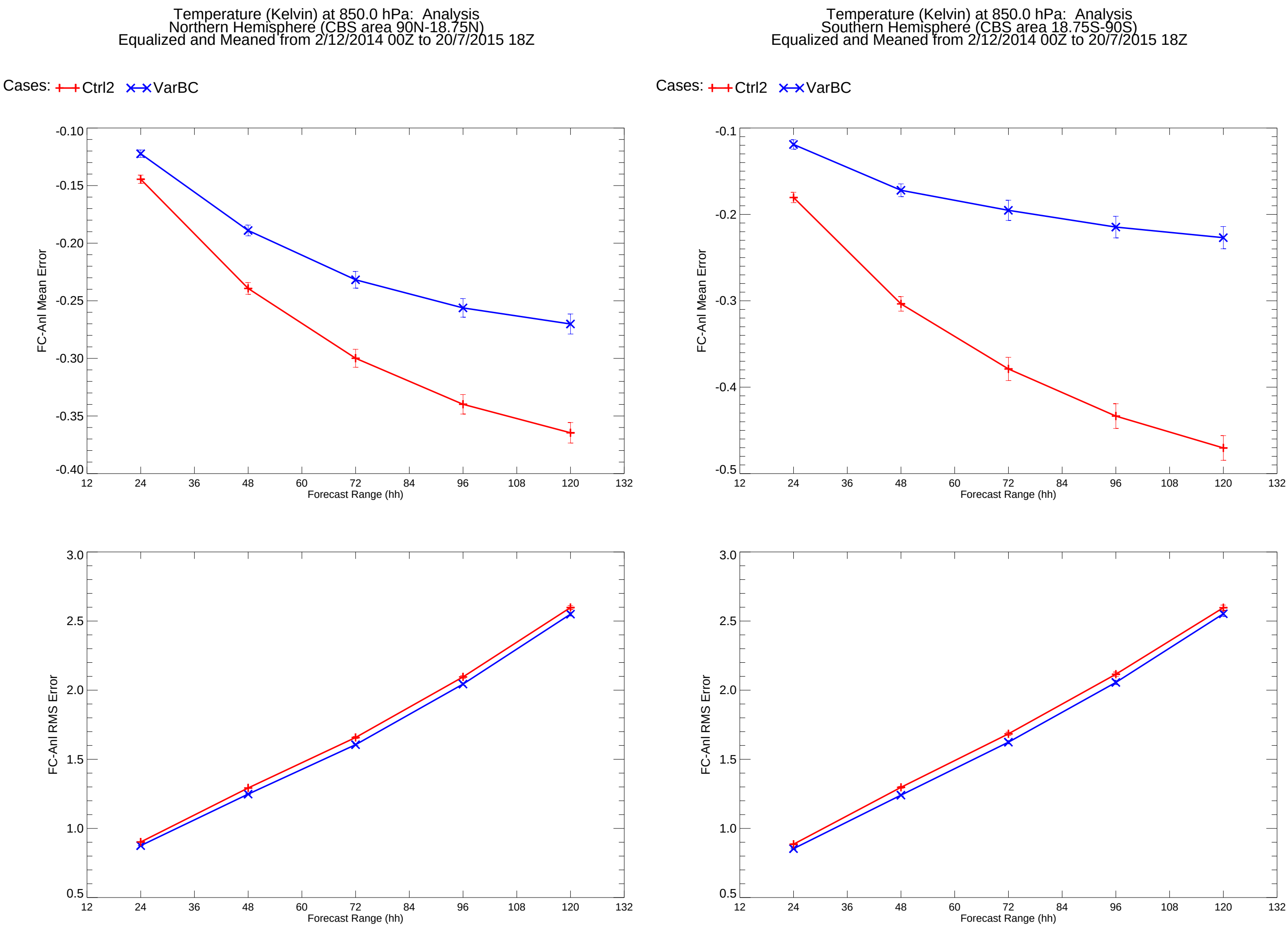


Figure 9: Verification of the temperature at 850 hPa against analysis for the long VarBC experiment and control. The forecast temperature tends to run colder with time. The spin-down is less with VarBC than in the control.

5 Conclusions

An initial implementation of variational bias correction has been extensively tested at the Met Office, including a 7.5 month trial and control. It is planned that VarBC will be implemented operationally in mid-March 2016. The VarBC scheme, in most respects, closely follows the implementation at other NWP centres, but several novel features have been introduced. These are: a harmonised adaption rate; a hybrid scan bias correction scheme comprising (static) spot-dependent offsets together with the use of Legendre Polynomial predictors to remove any residual, time-dependent biases; and a series of Fourier orbital predictors to correct for complex orbital biases.

The performance of VarBC has been consistently beneficial. VarBC produces cooler and drier analyses and a long-standing spin-down effect, in which initially high analysed tropospheric temperatures gradually relax back to climatological values, is much reduced in the VarBC experiments. The

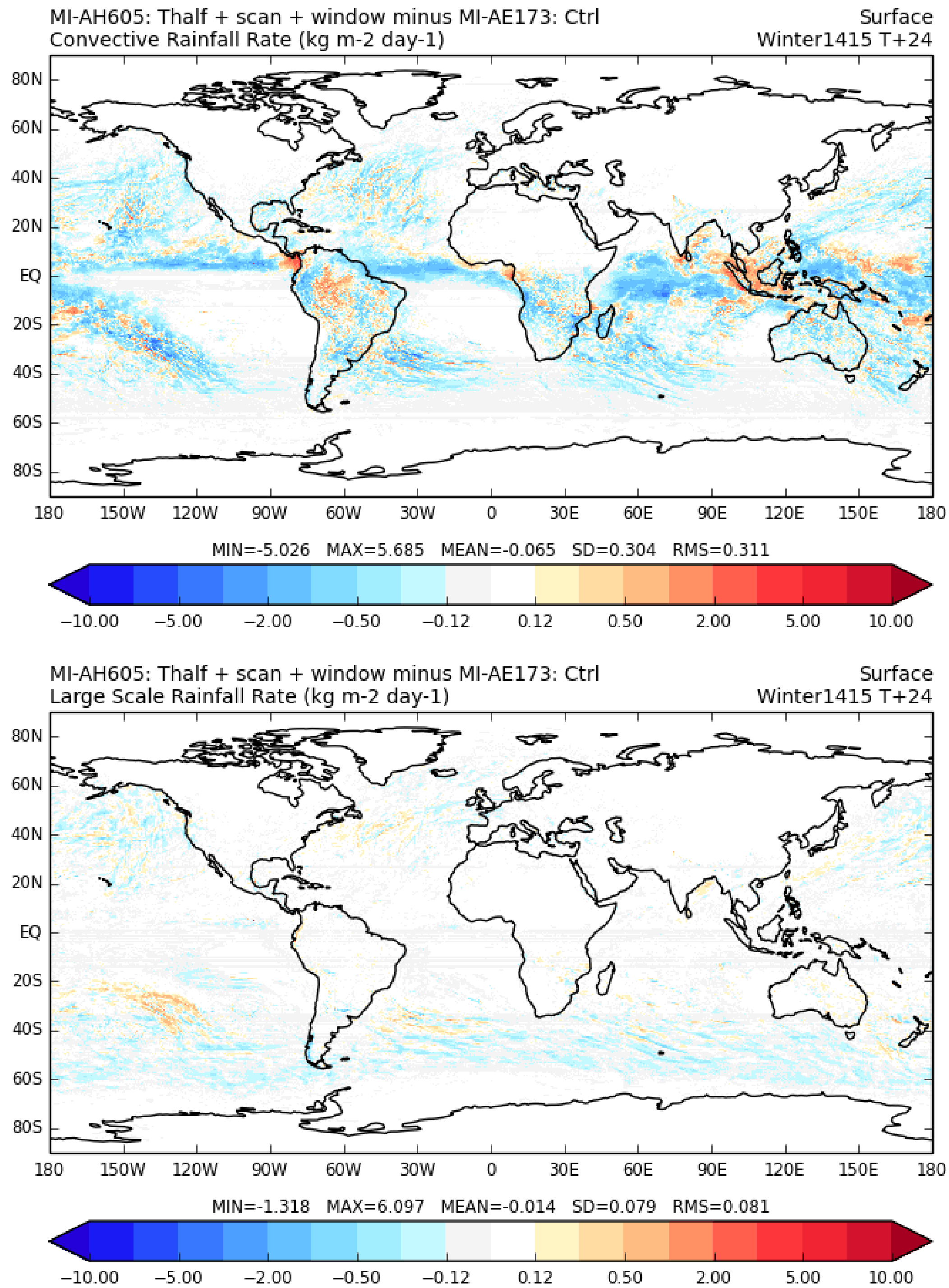


Figure 10: The change in convective rainfall rate and large scale rainfall rate at T+24 for the winter 2014/15 VarBC experiment minus the control. The ITCZ is generally less active in the VarBC experiment and there is a general reduction in rainfall in the mid-latitudes.

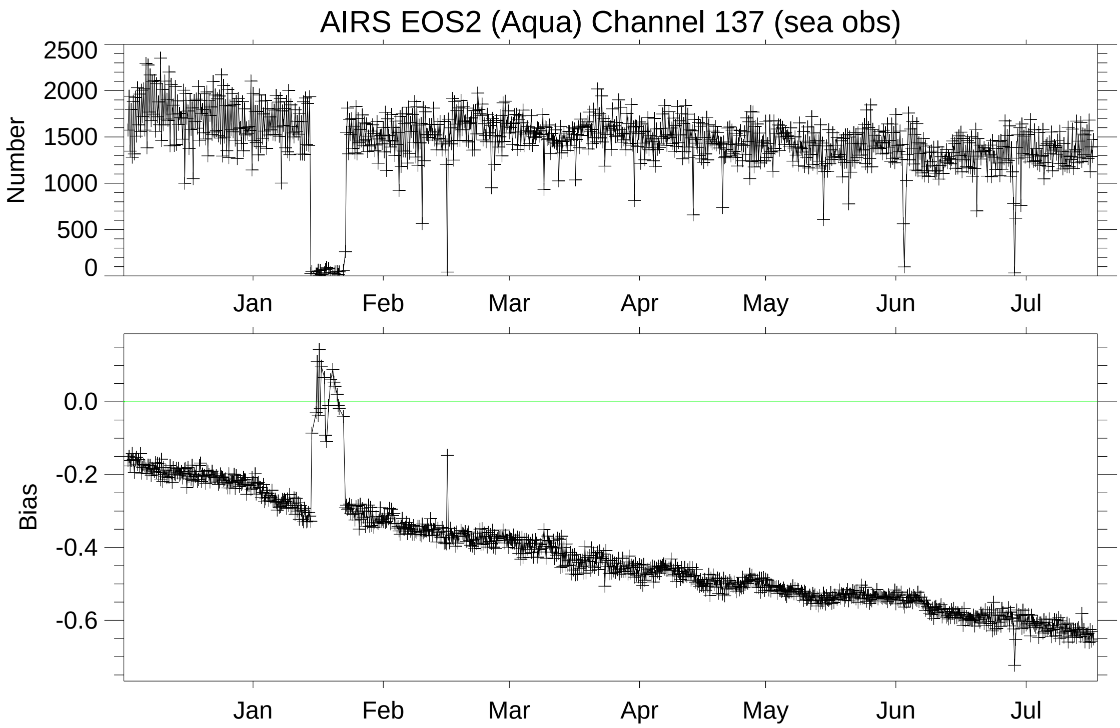


Figure 11: The applied bias for an AIRS window channel (absolute channel number 791 at 10.88 microns) from the 7.5 month VarBC stability trial. The applied bias drifts by around 0.5 K across 7.5 months.

verification of extra-tropical 500 hPa heights against analysis were improved by 5–10% in the VarBC experiments. Forecast wind fields and surface pressure fields are also improved. Both large scale and convective precipitation are reduced in the hours following a data assimilation cycle, relative to *operations-like* control experiments. In terms of diagnostics from the data assimilation system, short range forecast fits to almost all observations are significantly improved. For satellite radiances, in most cases the improvements are in the range 1–5%.

The priorities for future research and development work on VarBC are:

- Improving our understanding of the origin of the significant performance benefit brought by VarBC.
- Establishing a means of updating the bias correction applied to passive channels (those channels used for QC but not assimilated).
- Introduce a mechanism to selecting which observations influence the bias correction by surface or cloud type.
- Introduce a way to update the point-by-point scan bias correction that does not rely on the static scheme.
- Test VarBC actively in a regional model.

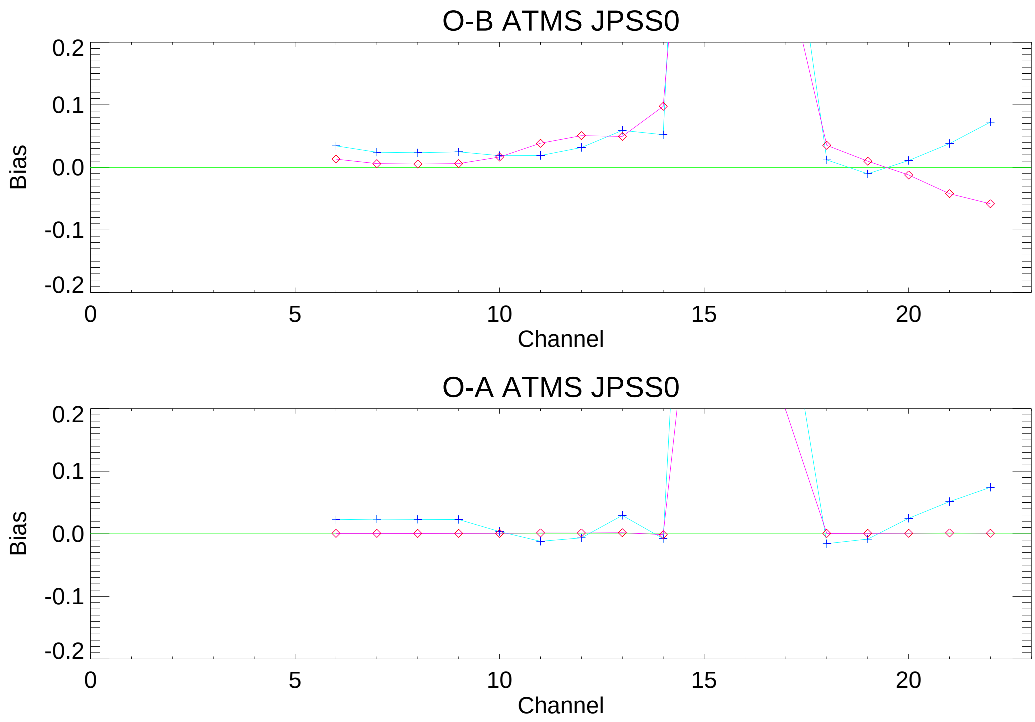


Figure 12: Mean O-B and O-A for ATMS for the 7.5 month control (blue/cyan) and VarBC trial (red/magenta). VarBC bias corrects to the analysis and the mean O-A is essentially zero for all bias corrected channels (ATMS-15 is not bias corrected).

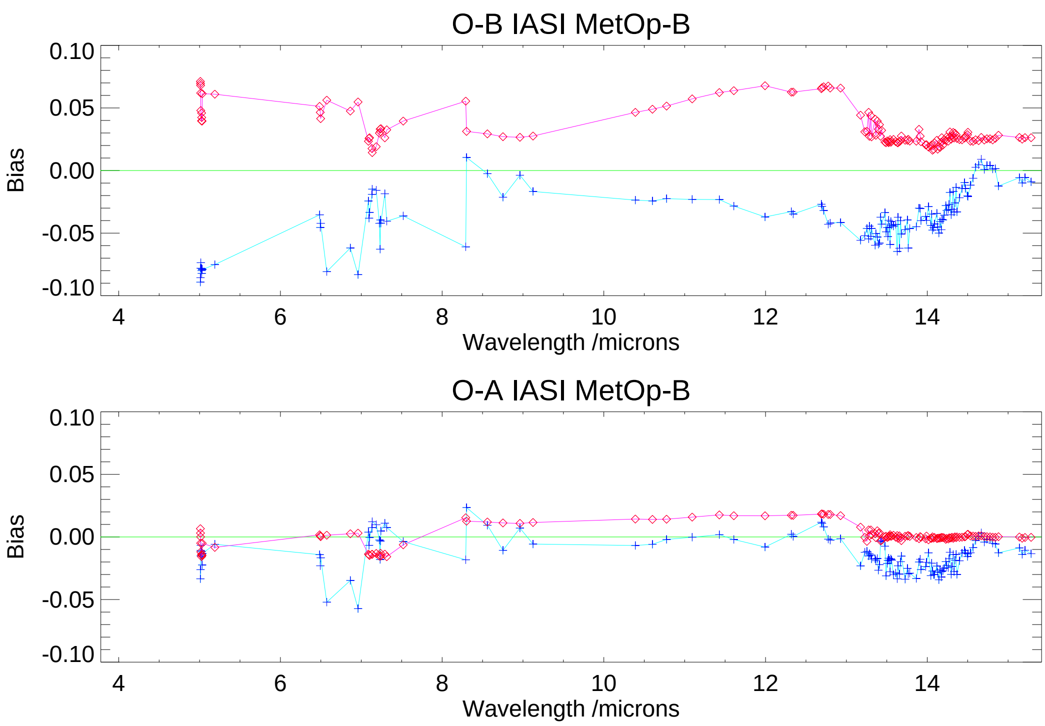


Figure 13: Mean O-B and O-A for IASI MetOp-B for the 7.5 month control (blue/cyan) and VarBC trial (red/magenta). The O-A for the long-wavelength CO_2 sounding channels is very close to zero, but there are small residuals of around 20 mK for the window channels.

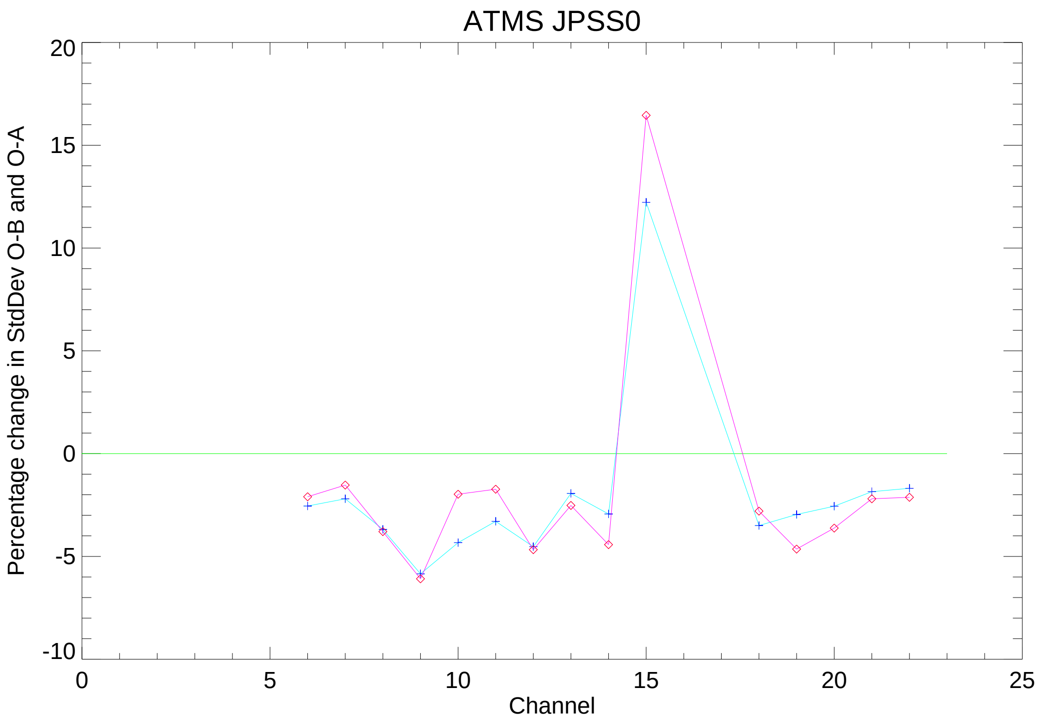


Figure 14: The percentage change in the standard deviation of O-B (blue/cyan) and O-A (red/magenta) for ATMS in the 7.5 month VarBC trial versus control. ATMS-15 is not bias corrected.

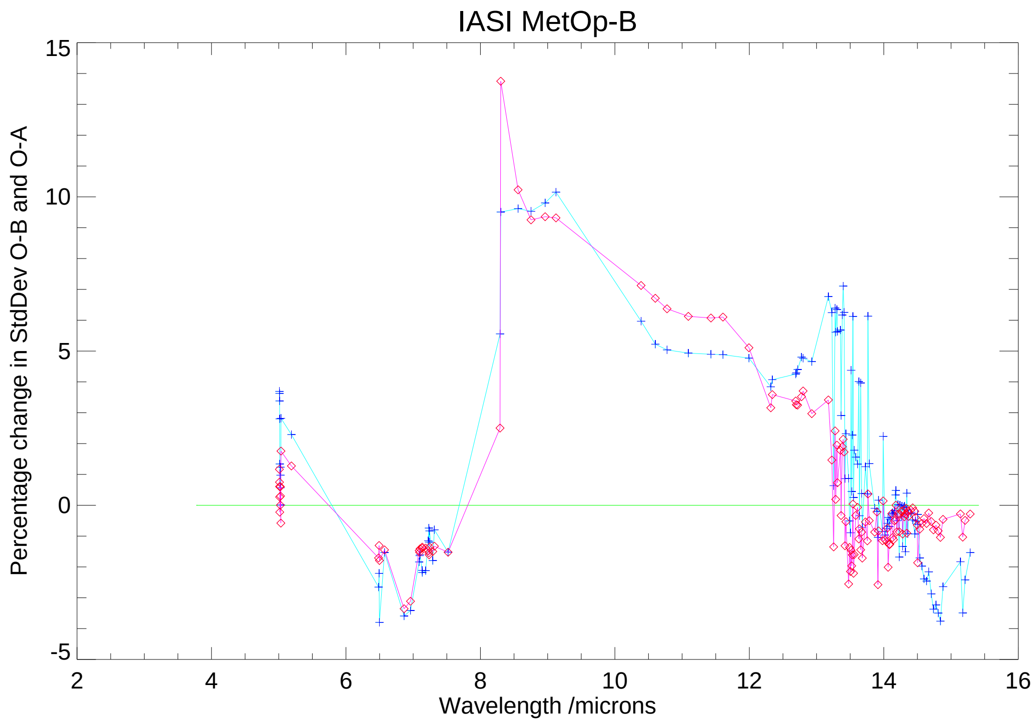


Figure 15: The percentage change in the standard deviation of O-B (blue/cyan) and O-A (red/magenta) for IASI MetOp-B in the 7.5 month VarBC trial versus control. The long-wavelength CO_2 sounding channels are well behaved but there is an increase in standard deviation for the window channels.

6 Acknowledgements

The development of the VarBC code at the Met Office was begun by Dingmin Li and largely finished by Andrew Lorenc, including documentation. Numerous people have assisted with the analysis and testing of VarBC, including William Bell, Fiona Smith, Masashi Ujiie, Anna Booton, and Peter Weston.

A Notation

$:=$	Assignment.
k	Observed datum subscript.
i	Predictor subscript.
I_k	The number of predictors used for datum k.
n	n^{th} assimilation cycle.
y_k^o	Observed value.
y_k	Forward modelled observed value.
$p_{k,i}$	Bias predictor value for datum k and predictor i.
β_i	Vector of bias coefficients for predictor i.
β_i^b	First guess (background) bias coefficients for predictor i.
β_i'	Corrections (increments) to β_i^b.
β_n	Single bias coefficient after n assimilation cycles. This definition overlaps with β_i.
$V_{(\beta_i)}$	The value from the diagonal background error for the bias coefficient β_i.

B The bias adaption rate

The weighted mean of two numbers β_b and β_o with errors σ_b and σ_o is usually calculated as:

$$\overline{\beta} = \frac{\dfrac{\beta_b}{\sigma_b^2} + \dfrac{\beta_o}{\sigma_o^2}}{\dfrac{1}{\sigma_b^2} + \dfrac{1}{\sigma_o^2}} \tag{7}$$

If $\sigma_b^2 = \frac{m}{N}\sigma_o^2$ then this simplifies to:

$$\overline{\beta} = \frac{N}{N+m}\beta_b + \frac{m}{N+m}\beta_o \tag{8}$$

In VarBC the background error for the bias coefficients is chosen so that the weight of the prior is N_{bgerr}/m times the weight of the observations when determining the analysed β, where N_{bgerr} is a constant chosen by the user and m is the number of observations that include a particular channel in the assimilation cycle.

In the special case where the data assimilation control vector only contains increments to the

VarBC bias coefficients (no atmospheric variables) then after the n^{th} assimilation cycle the coefficient would be:

$$\beta_n = \frac{N_{bgerr}}{N_{bgerr}+m}\beta_{n-1} + \frac{m}{N_{bgerr}+m}\beta_{\text{best}} \tag{9}$$

where β_{n-1} is the previous bias coefficient, and β_{best} is the value of the coefficient that minimises the observation penalty. Note that if $m = N_{bgerr}$ then the difference between β_n and β_{best} will halve each assimilation cycle (this follows directly from the how N_{bgerr} is defined). If the initial bias coefficient was β_0 then after n assimilation cycles:

$$\beta_n = \left(\frac{N_{bgerr}}{N_{bgerr}+m}\right)^n \beta_0 + \left[1 - \left(\frac{N_{bgerr}}{N_{bgerr}+m}\right)^n\right]\beta_{\text{best}} \tag{10}$$

This represents an exponential decay from an initial value β_0 towards the best fit value β_{best}. The difference from the best fit value will halve when

$$\left(\frac{N_{bgerr}}{N_{bgerr}+m}\right)^n = \frac{1}{2} \tag{11}$$

Re-arranging for the N_{bgerr} that will lead to the difference halving in n assimilation cycles:

$$N_{bgerr} = m\left(\frac{1}{2^{\frac{1}{n}}-1}\right) \tag{12}$$

At the Met Office the N_{bgerr} is set for each bias predictor as:

$$N_{bgerr} = \texttt{MAX}(m_{avg}, M_{min})\left(\frac{1}{2^{\frac{1}{n}}-1}\right) \tag{13}$$

where m_{avg} is the expected number of observations in a data assimilation cycle that contain a given channel, M_{min} is a fixed number chosen by the user (e.g. 1000 observations), and n is set by the user to be the desired residual bias halving time in units of data assimilation cycles (e.g. 8 DA cycles). At the Met Office a running estimate of m_{avg} is stored in the VarBC coefficients file and automatically updated each DA cycle.

In equation 13 the minimum possible value for N_{bgerr} is determined by M_{min} and n. VarBC has been tested at the Met Office with $M_{min} = 1000$ and $n = 8$ (there are 4 data assimilation cycles per day at the Met Office so this corresponds to a bias coefficient halving time of 2 days), resulting in a *minimum* N_{bgerr} of about 11,000. M_{min} is an important safety mechanism in the system because in the case of a period of low data volumes then the estimate of m_{avg} would drift lower and the bias correction could end up being based on dangerously low numbers of observations. For channels where $m_{avg} > M_{min}$ (normally true for the vast majority of channels) then the N_{bgerr} will be set correspondingly larger such that the bias halving time is determined by n (8 DA cycles or 2 days), resulting in a harmonised adaption rate across channels. Harmonising the bias adaption rate is important at the Met Office because the data assimilation window is only 6 hours. In 6 hours a polar orbiting satellite has not sampled the full globe and has not sampled the diurnal variation for the parts it has covered, and therefore may not have sampled the full range of biases and bias predictors it will later encounter. The bias halving time enables the retention of bias information from previous DA cycles, even for data-rich channels. The bias halving time formulation may be useful for running VarBC actively in a regional model, where it could prove beneficial to average out diurnal variations in data volume and bias.

Equation 9 is for the special case that the control vector only contains the bias coefficients. In the case where the control vector consists of atmospheric variables as well as the bias coefficients, then equation 10 is an *overestimate* of how quickly the bias coefficients will adapt. The adaption will be slowest where the analysis is most able to mold itself to the form of the bias. This will happen where observations errors are small and the background error is large.

References

[Auligné et al., 2007] Auligné, T. et al. (2007). Adaptive bias correction for satellite data in a numerical weather prediction system. *Q. J. R. Meteorol. Soc.*, 133:631–642. doi: 10.1002/qj.56.

[Bell, 2008] Bell, W. (2008). The assimilation of ssmis radiances in numerical weather prediction models. IEEE Transactions on Geoscience and Remote Sensing, Vol 45.

[Booton, 2014] Booton, A. (2014). An improved bias correction for ssmis. Proceedings of the International TOVS Study Conference-19, Jeju Island, South Korea.

[Dee, 2004] Dee, D. (2004). Variational bias correction of radiance data in the ECMWF system. In *Proceedings of ECMWF Workshop on Assimilation of high spectral resolution sounders in NWP*. URL: `http://old.ecmwf.int/publications/library/ecpublications/_pdf/workshop/2004/ws_AIRS_dee.pdf`.

[Harris and Kelly, 2001] Harris, B. and Kelly, G. (2001). A satellite radiance bias correction scheme for data assimilation. *Q. J. R. Meteorol. Soc.*, 127:1453–1468.