Physical Simultaneous Retrieval of Emissivity Spectrum and Thermodynamical parameters: A case study for desert soils

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Basic methodological steps to retrieve surface emissivity

Step 1: Represent emissivity with a Fourier cosine truncated series to lower it dimensionality below that of the IASI spectrum.

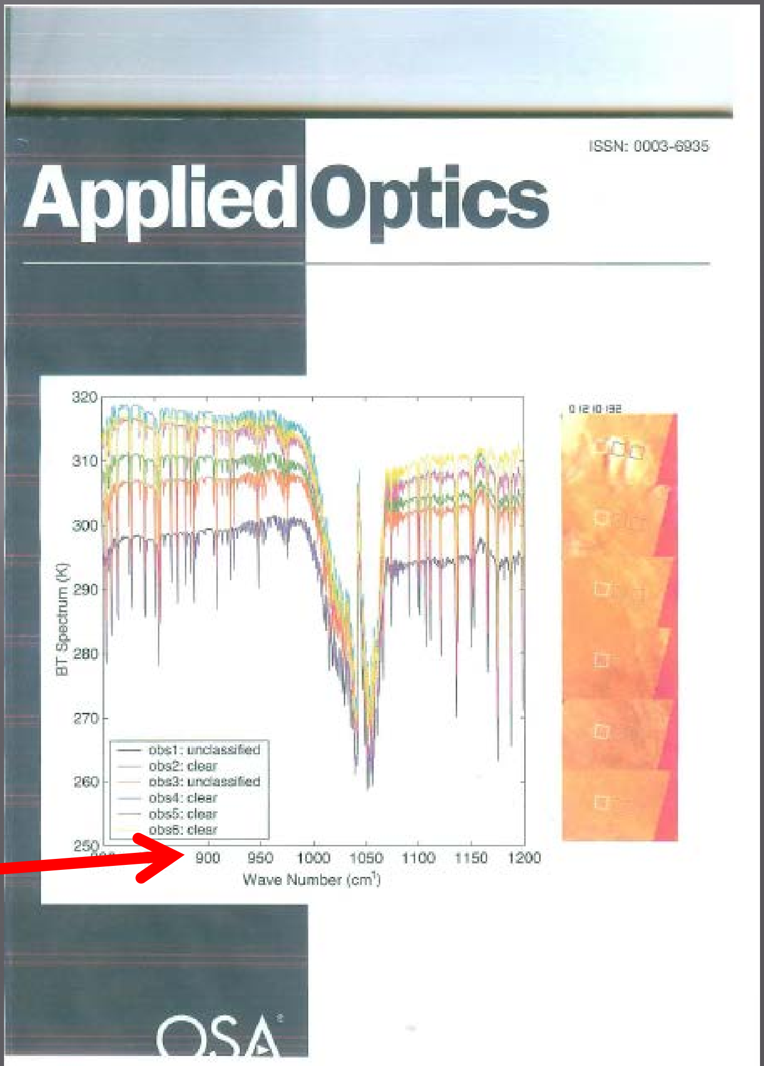
Step 2: Constrain the retrieval with Laboratory measurements

Step 3: Balance between Atmospheric and Emissivity Constraints with a 2-Dimensional L-curve criterion

Basic methodological steps to retrieve surface emissivity

Step 2: Constrain the retrieval with Laboratory measurements

- Guido Masiello, Carmine Serio, and Vincenzo Cuomo, "Exploiting Quartz Spectral Signature for the Detection of Cloud-Affected Satellite Infrared Observations over African Desert Areas," Appl. Opt. **43**, 2305-2315 (2004)



Background Retrieval Methodology: the retrieval problem is formulated within the context of optimal estimation

$$(R-F(v))^T S_\varepsilon^{-1}(R-F(v))+(v-v_a)^T S_a^{-1}(v-v_a)$$

Linearize

$$(y-Kx)^T S_\varepsilon^{-1}(y-Kx)+(x-x_a)^T S_a^{-1}(x-x_a)$$

$$y=R-F(v_o)$$

$$x=v-v_o$$

$$x_a=v_a-v_o$$

$$K=\left(\frac{\partial F}{\partial v}\right)_{v=v_o}$$

How we modify the Physics to include surface emissivity

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- Use the ***logit*** transform to map emissivity from the interval [0,1] to the range [-∞, +∞]

$$y(i) = \text{logit}(\varepsilon(i)) = \log\left(\frac{\varepsilon(i)}{1-\varepsilon(i)}\right) \quad i = 1,\ldots,N_{ch}; \quad N_{ch} = \text{IASI channels}$$

- which has the inverse

$$\varepsilon(i) = \frac{\exp(y(i))}{1+\exp(y(i))} \quad i = 1,\ldots,N_{ch}(i)$$

To retrieve for emissivity, the given radiance *R*(*i*), with *i* the channel, is first linearized also with respect to the function *y*(*i*), that is we consider in the inverse problem also a linear term of the type

$$\frac{\partial R(i)}{\partial y(i)}(y(i)-y_o(i)) \qquad (3)$$

with $y_o(i)$ a suitable first guess. The derivative term can be easily computed when we consider the dependence of the radiance on the surface term,

$$\varepsilon(i)\tau_o(i)B(T_g)$$

where $\tau_o(i)$ is the total transmittance at channel *i*, and $B(T_g)$ is the Planck function computed at the ground-surface temperature T_g. We have for the derivative,

$$\frac{\partial R(i)}{\partial y(i)}=\frac{\partial R(i)}{\partial \varepsilon(i)}\frac{\partial \varepsilon(i)}{\partial y(i)}=\frac{\partial R(i)}{\partial \varepsilon(i)}\left(\frac{\partial y(i)}{\partial \varepsilon(i)}\right)^{-1}=\frac{\partial R(i)}{\partial \varepsilon(i)}\varepsilon(i)\left(1-\varepsilon(i)\right)=\tau_o(i)B(T_g)\varepsilon(i)\left(1-\varepsilon(i)\right)$$

Continue………….
Second, we develop the function in a truncated cosine series

$$y(i)=\sum_{k=1}^{N_{cut}} w(k)c(k)\cos\frac{\pi(2i-1)(k-1)}{2N_{ch}}$$

$$w(k)=\begin{cases}\sqrt{\dfrac{1}{N_{ch}}} & k=1\\ \sqrt{\dfrac{2}{N_{ch}}} & k=2,\ldots,N_{cut}\end{cases}$$

where, $N_{cut} << N_{ch}$. The Fourier coefficients, $c(k)$ can be obtained by,

$$c(k)=w(k)\sum_{i=1}^{N_{ch}} y(k)\cos\frac{\pi(2i-1)(k-1)}{2N_{ch}} \quad k=1,\ldots,N_{ch}$$

$$w(k)=\begin{cases}\sqrt{\dfrac{1}{N_{ch}}} & k=1\\ \sqrt{\dfrac{2}{N_{ch}}} & k=2,\ldots,N_{ch}\end{cases}$$

Continue............

and Inserting the truncated cosine transform within the linear term (3), we get

$$\frac{\partial R(i)}{\partial y(i)}\left(\sum_{k=1}^{N_{cut}} w(k)c(k)\cos\frac{\pi(2i-1)(k-1)}{2N_{ch}} - \sum_{k=1}^{N_{cut}} w(k)c_o(k)\cos\frac{\pi(2i-1)(k-1)}{2N_{ch}}\right)$$

which defining the jacobian matrix,

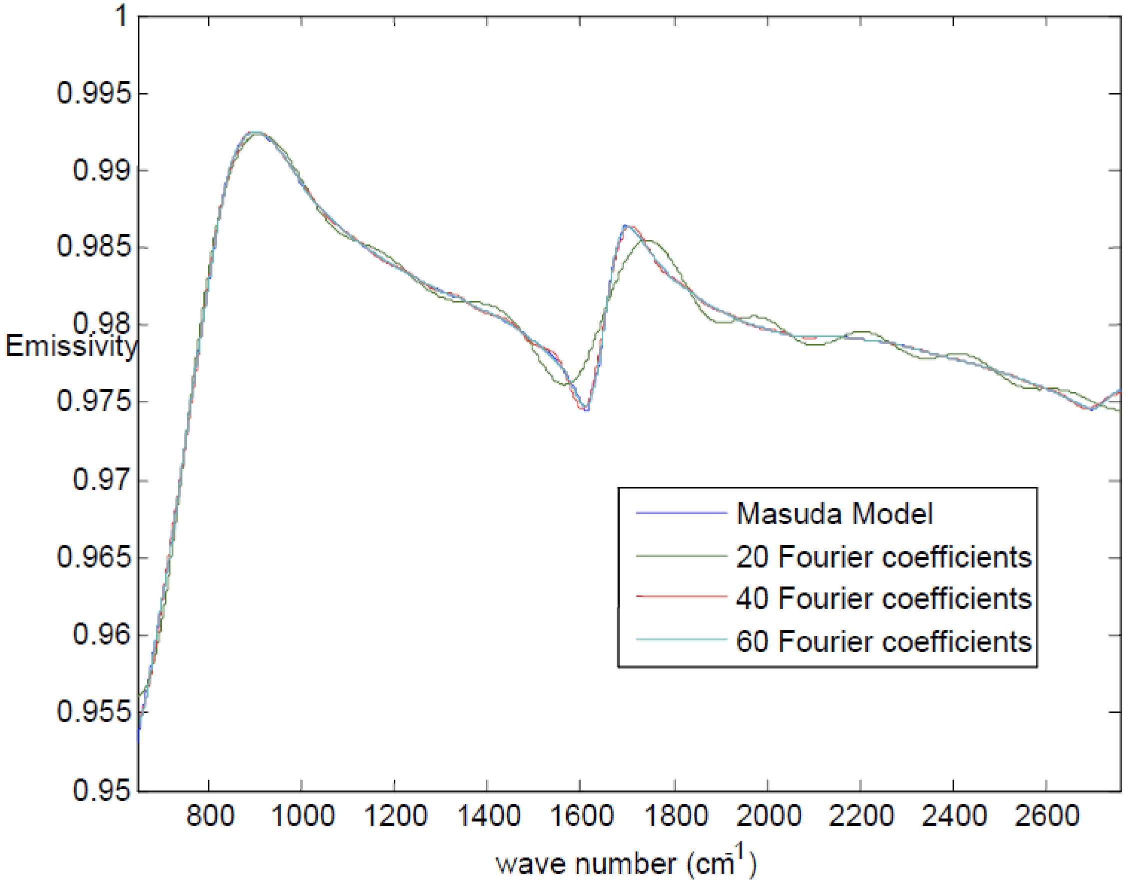
$$A_{ik} = \frac{\partial R(i)}{\partial y(i)} w(k)\cos\frac{\pi(2i-1)(k-1)}{2N_{ch}}, \quad i=1,\ldots,N_{ch};\, k=1,\ldots,N_{cut}$$

allows us to rewrite the linear term (2) in a matrix form,

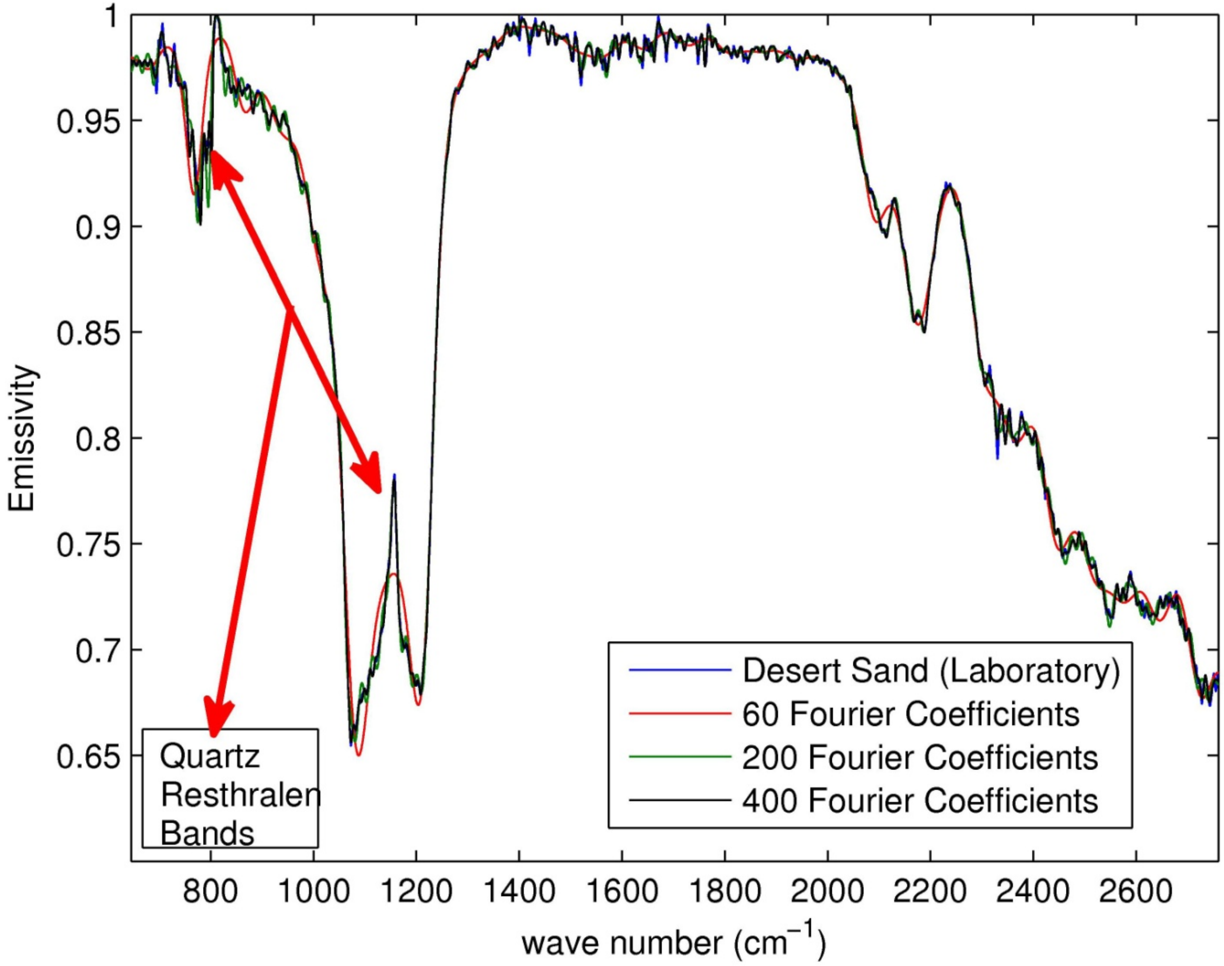
$$\mathbf{A}(\mathbf{c}-\mathbf{c}_o) \qquad (4)$$

Which is suitable for inversion. Note that in (4) the cosine coefficients vectors have size N_{cut}, and the matrix **A** has size $N_{ch} \times N_{cut}$..

The Fourier transform allows us to work in terms of spectral resolution, exactly the way we deal with this concept. For sea emissvity, 60 Fourier Coefficients are enough



…but, if we want to resolve the Quartz Resthralen bands in desert soil we need more than 200 Fourier Coefficients



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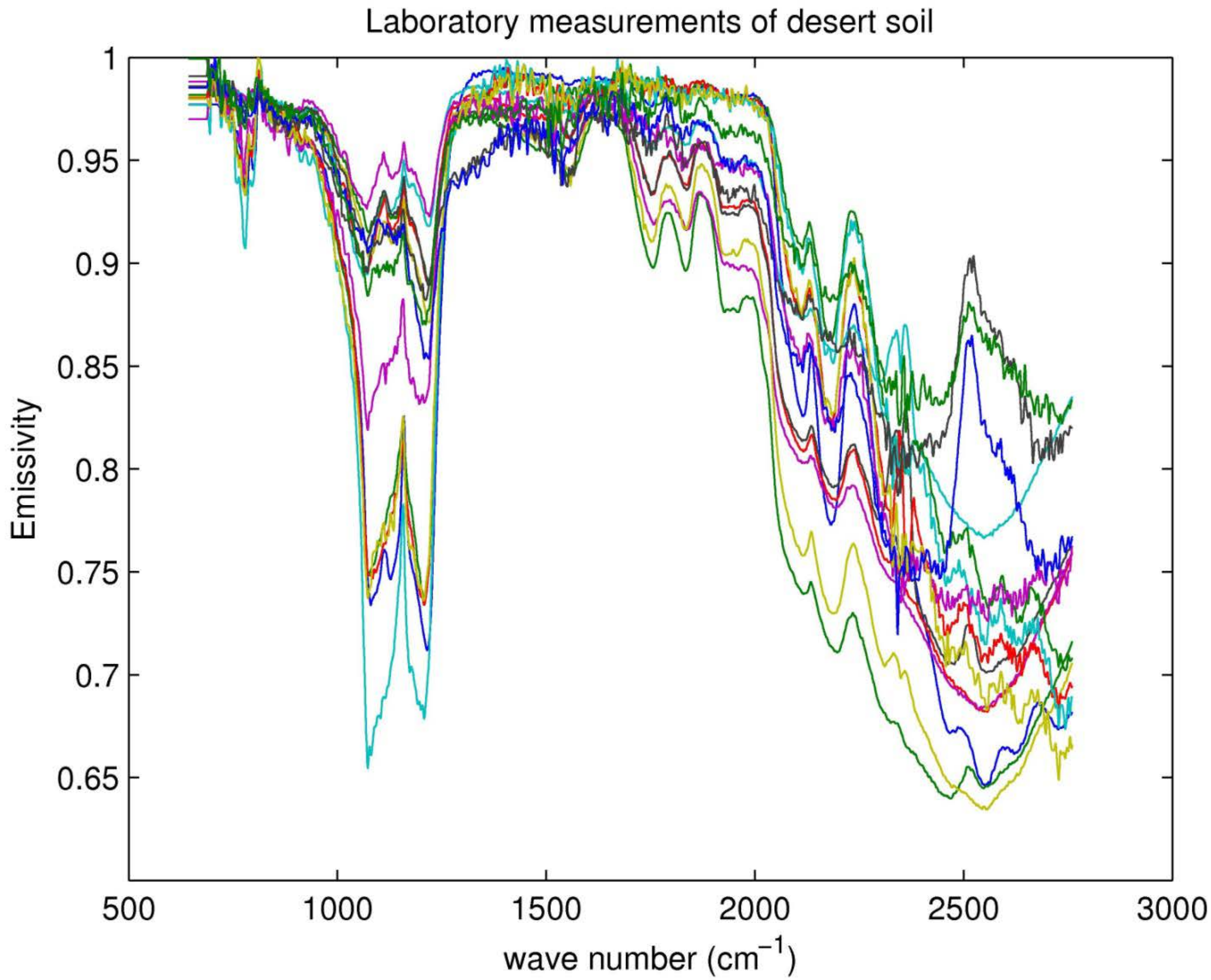
From the size of background constraint we introduce information from laboratory measurements

Laboratory emissivity is used for the background, mean and covariance matrix assumed to be diagonal (ASTER-Salisbury data base)



The whole covariance matrix for atmospheric parameters and emissivity is built up in block-diagonal matrix

Atmosphere

$$\mathbf{S}_a = \begin{pmatrix} \gamma_1 \mathbf{S}_a^{\mathrm{Atm}} & 0 \\ 0 & \gamma_2 \mathbf{S}_a^{\mathrm{Emis}} \end{pmatrix}$$

Emissivity

γ_1 and γ_2 can be optimized to balance between the two terms. Balancing is obtained with an original and fully analytical implementation of a 2-D L-curve criterion

Fully 2-D L-curve method, outline of the mathematics involved

$$L(\gamma_1,\gamma_2) =_{def} \begin{cases} z = & \varphi(\gamma_1,\gamma_2) = & (\mathbf{G}\hat{\mathbf{u}}-\mathbf{y})^t(\mathbf{G}\hat{\mathbf{u}}-\mathbf{y}) \\ x = & \psi(\gamma_1,\gamma_2) = & \hat{\mathbf{u}}^t\hat{\mathbf{I}}_{\gamma_1}\hat{\mathbf{u}} \\ y = & \omega(\gamma_1,\gamma_2) = & \hat{\mathbf{u}}^t\hat{\mathbf{I}}_{\gamma_2}\mathbf{u} \end{cases} \qquad (1)$$

The Gaussian curvature is given [?] by

$$\kappa(\gamma_1,\gamma_2) = \frac{\mid \det P \mid}{w^4} \qquad (2)$$

with

$$w^2 = 1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial x}\right)^2 \qquad (3)$$

and where $\mid \det P \mid$ is the absolute value of the determinant of the matrix, $\mathbf{P}$,whose elements, P_{ij} are

$$P_{ij} = \frac{\partial^2 z}{\partial x_i \partial x_j} \qquad (4)$$

with $i, j = 1, 2$ and $x_1 = x$, $x_2 = y$. The derivatives above can be obtained by a transformation of partial derivatives of φ, ψ, ω with respect the two regularization parameters, γ_1, γ_2. Introducing the notation,

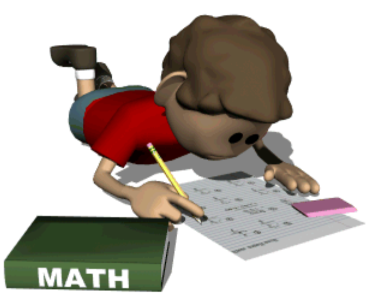
$$\begin{cases} X_i = & \frac{\partial X}{\partial \gamma_i}; & i = 1,2 \\ X_{ij} = & \frac{\partial^2 X}{\partial \gamma_i \partial \gamma_j}; & i,j = 1,2 \end{cases} \qquad (5)$$

to indicate the derivative of a given function X with respect to γ_1 and γ_2, we have

$$\begin{cases} \frac{\partial z}{\partial x} = & g = & \frac{\varphi_1}{\psi_1} + \frac{\varphi_2}{\psi_2} \\ \frac{\partial z}{\partial y} = & g^* = & \frac{\varphi_1}{\omega_1} + \frac{\varphi_2}{\omega_2} \end{cases} \qquad (6)$$

And for the second derivatives,

$$\begin{cases} \frac{\partial^2 z}{\partial x^2} = & \frac{g_1}{\psi_1} + \frac{g_2}{\psi_2}; & \frac{\partial^2 z}{\partial x \partial y} = & \frac{g_1^*}{\psi_1} + \frac{g_2^*}{\psi_2} \\ \frac{\partial^2 z}{\partial y^2} = & \frac{g_1^*}{\omega_1} + \frac{g_2^*}{\omega_2}; & \frac{\partial^2 z}{\partial y \partial x} = & \frac{g_1}{\omega_1} + \frac{g_2}{\omega_2} \end{cases} \qquad (7)$$



with

$$\left\{\begin{array}{ll} g_1 = & \frac{\varphi_{11}}{\psi_1} - \frac{\varphi_1\psi_{11}}{\psi_1^2} + \frac{\varphi_{12}}{\psi_2} - \frac{\varphi_2\psi_{12}}{\psi_2^2} \\ g_2 = & \frac{\varphi_{22}}{\psi_2} - \frac{\varphi_2\psi_{22}}{\psi_2^2} + \frac{\varphi_{21}}{\psi_1} - \frac{\varphi_1\psi_{21}}{\psi_1^2} \\ g_1^* = & \frac{\varphi_{11}}{\omega_1} - \frac{\varphi_1\omega_{11}}{\omega_1^2} + \frac{\varphi_{12}}{\omega_2} - \frac{\varphi_2\omega_{12}}{\omega_2^2} \\ g_2^* = & \frac{\varphi_{22}}{\omega_2} - \frac{\varphi_2\omega_{22}}{\omega_2^2} + \frac{\varphi_{21}}{\omega_1} - \frac{\varphi_1\omega_{21}}{\omega_1^2} \end{array}\right. \tag{8}$$

The notation X_i and X_{ij} for the first and second derivatives allows us to deal with these quantities as component of a vector and a matrix, respectively. This greatly simplifies the software implementation of an algorithm to compute the Gaussian curvature for each given couples (γ_1, γ_2).

At this point we need a scheme to compute the derivatives of φ, ψ, ω. These can be obtained by a direct differentiation of the parametric surface of Eq. 2. Continuing to use the simplified notation also for the derivatives of the vector solution, $\hat{\mathbf{u}}$, we have for thr derivatives of φ,

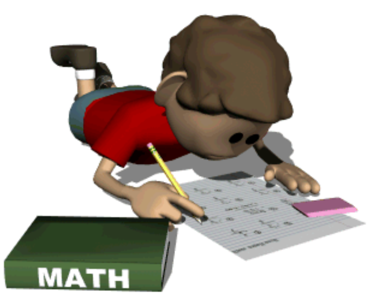
$$\left\{\begin{array}{ll} \varphi_1 = & (\mathbf{G}\hat{\mathbf{u}}_1)^t(\mathbf{G}\hat{\mathbf{u}} - \mathbf{y}) + (\mathbf{G}\hat{\mathbf{u}} - \mathbf{y})^t(\mathbf{G}\hat{\mathbf{u}}_1) \\ \varphi_2 = & (\mathbf{G}\hat{\mathbf{u}}_2)^t(\mathbf{G}\hat{\mathbf{u}} - \mathbf{y}) + (\mathbf{G}\hat{\mathbf{u}} - \mathbf{y})^t(\mathbf{G}\hat{\mathbf{u}}_2) \\ \varphi_{11} = & (\mathbf{G}\hat{\mathbf{u}}_{11})^t(\mathbf{G}\hat{\mathbf{u}} - \mathbf{y}) + 2(\mathbf{G}\hat{\mathbf{u}}_1)^t(\mathbf{G}\hat{\mathbf{u}}_1) + (\mathbf{G}\hat{\mathbf{u}} - \mathbf{y})^t(\mathbf{G}\hat{\mathbf{u}}_{11}) \\ \varphi_{22} = & (\mathbf{G}\hat{\mathbf{u}}_{22})^t(\mathbf{G}\hat{\mathbf{u}} - \mathbf{y}) + 2(\mathbf{G}\hat{\mathbf{u}}_2)^t(\mathbf{G}\hat{\mathbf{u}}_2) + (\mathbf{G}\hat{\mathbf{u}} - \mathbf{y})^t(\mathbf{G}\hat{\mathbf{u}}_{22}) \\ \varphi_{12} = & (\mathbf{G}\hat{\mathbf{u}}_{12})^t(\mathbf{G}\hat{\mathbf{u}} - \mathbf{y}) + (\mathbf{G}\hat{\mathbf{u}}_2)^t(\mathbf{G}\hat{\mathbf{u}}_1) + (\mathbf{G}\hat{\mathbf{u}}_1)^t(\mathbf{G}\hat{\mathbf{u}}_2) + (\mathbf{G}\hat{\mathbf{u}} - \mathbf{y})^t(\mathbf{G}\hat{\mathbf{u}}_{12}) \\ \varphi_{21} = & (\mathbf{G}\hat{\mathbf{u}}_{21})^t(\mathbf{G}\hat{\mathbf{u}} - \mathbf{y}) + (\mathbf{G}\hat{\mathbf{u}}_1)^t(\mathbf{G}\hat{\mathbf{u}}_2) + (\mathbf{G}\hat{\mathbf{u}}_2)^t(\mathbf{G}\hat{\mathbf{u}}_1) + (\mathbf{G}\hat{\mathbf{u}} - \mathbf{y})^t(\mathbf{G}\hat{\mathbf{u}}_{21}) \end{array}\right. \tag{9}$$

For the derivatives of ψ, we have

$$\left\{\begin{array}{ll} \psi_1 = & \hat{\mathbf{u}}_1^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}} + \hat{\mathbf{u}}^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_1 \\ \psi_2 = & \hat{\mathbf{u}}_2^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}} + \hat{\mathbf{u}}^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_2 \\ \psi_{11} = & \hat{\mathbf{u}}_{11}^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}} + 2\hat{\mathbf{u}}_1^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_1 + \hat{\mathbf{u}}^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_{11} \\ \psi_{22} = & \hat{\mathbf{u}}_{22}^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}} + 2\hat{\mathbf{u}}_2^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_2 + \hat{\mathbf{u}}^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_{22} \\ \psi_{12} = & \hat{\mathbf{u}}_{12}^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}} + \hat{\mathbf{u}}_2^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_1 + \hat{\mathbf{u}}_1^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_2 + \hat{\mathbf{u}}^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_{12} \\ \psi_{21} = & \hat{\mathbf{u}}_{21}^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}} + \hat{\mathbf{u}}_1^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_2 + \hat{\mathbf{u}}_2^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_2 + \hat{\mathbf{u}}^t\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_{21} \end{array}\right. \tag{10}$$

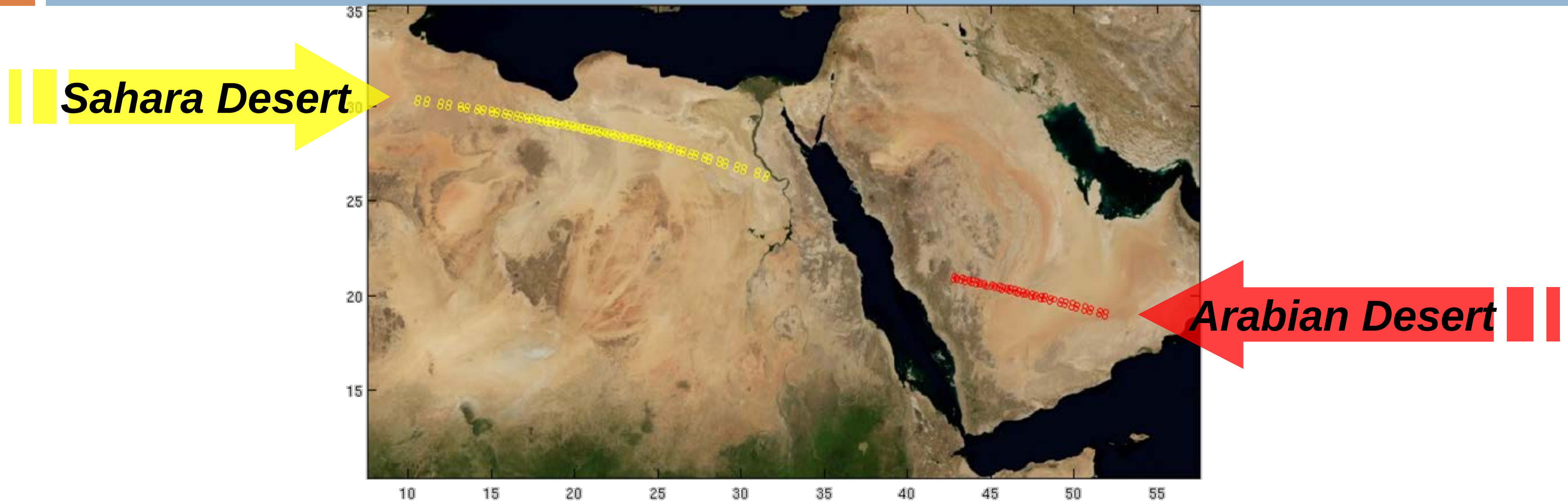
Because of the symmetry of two terms ψ and ω, the derivatives of ω are obtained by replacing $\mathbf{I}_{\gamma_1}$ with $\mathbf{I}_{\gamma_2}$ in Eq. 10.

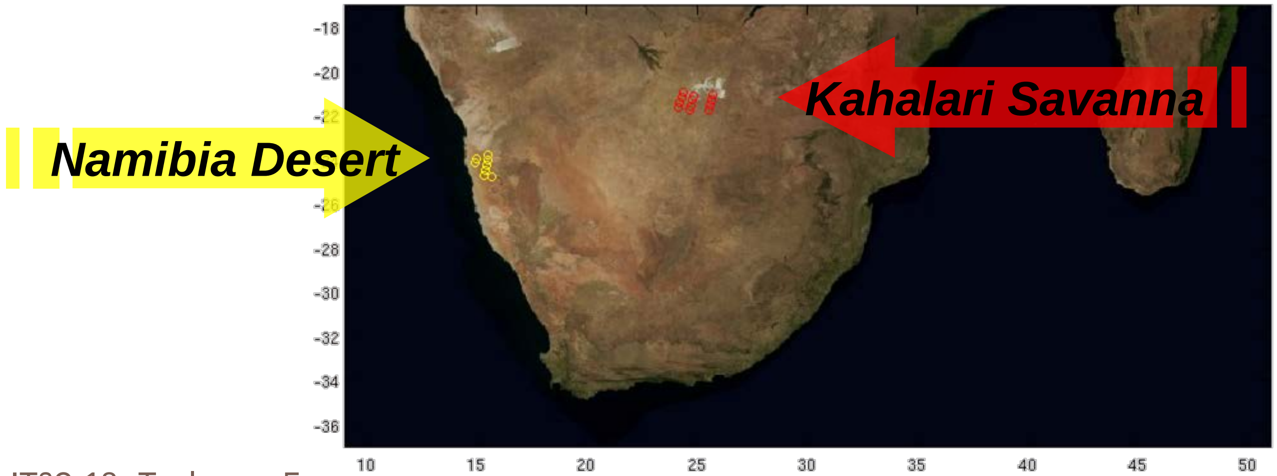
Finally, it is seen that all the computations above depend on the partial derivatives of the $\hat{\mathbf{u}}$. These can be easily obtained by differentiation of Eq. ??. We have



$$\left\{\begin{array}{ll} \mathbf{A}\hat{\mathbf{u}}_1 = & -\mathbf{I}_{\gamma_1}\hat{\mathbf{u}} \\ \mathbf{A}\hat{\mathbf{u}}_2 = & -\mathbf{I}_{\gamma_2}\hat{\mathbf{u}} \\ \mathbf{A}\hat{\mathbf{u}}_{11} = & -2\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_1 \\ \mathbf{A}\hat{\mathbf{u}}_{22} = & -2\mathbf{I}_{\gamma_2}\hat{\mathbf{u}}_2 \\ \mathbf{A}\hat{\mathbf{u}}_{12} = & -\mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_2 - \mathbf{I}_{\gamma_2}\hat{\mathbf{u}}_1 \\ \mathbf{A}\hat{\mathbf{u}}_{21} = & -\mathbf{I}_{\gamma_2}\hat{\mathbf{u}}_1 - \mathbf{I}_{\gamma_1}\hat{\mathbf{u}}_2 \end{array}\right. \tag{11}$$

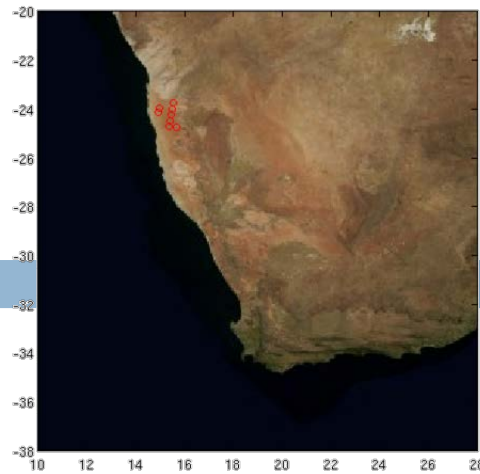
Retrieval exercise over desert areas

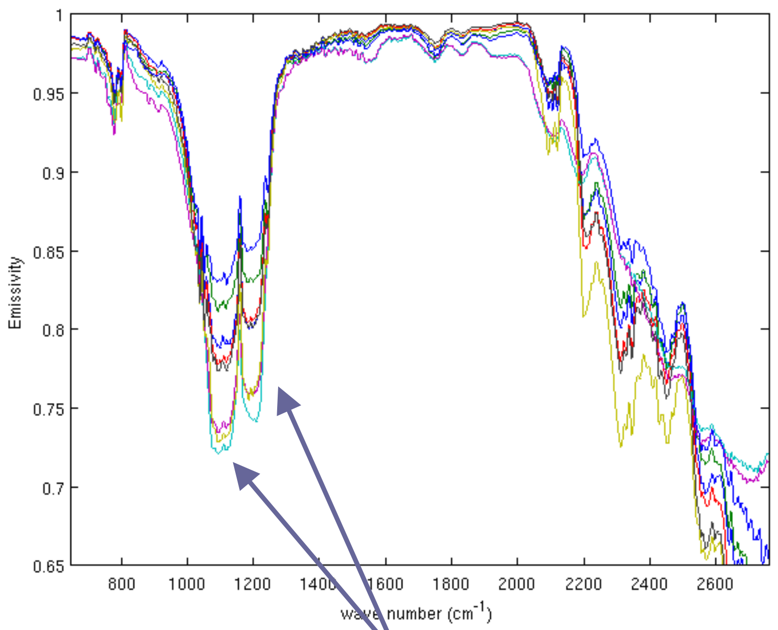


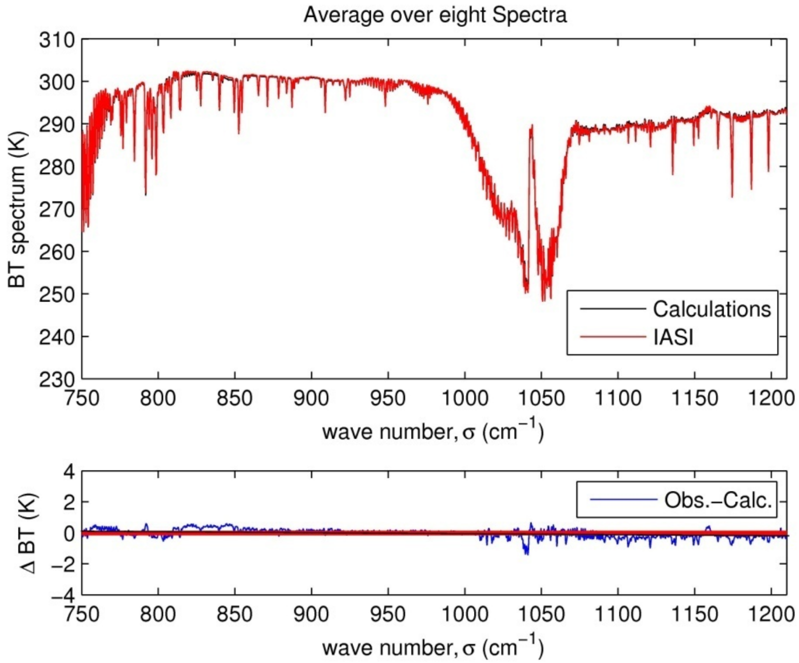


- July 22, 2007,
  - 6:45 (Arabia)
  - 8:35 (Other)

Results: Namibia desert

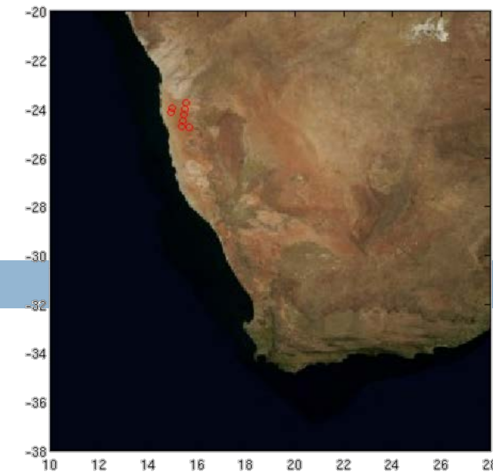


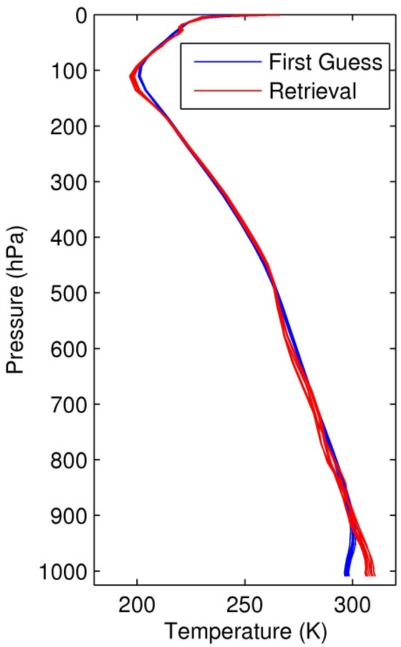


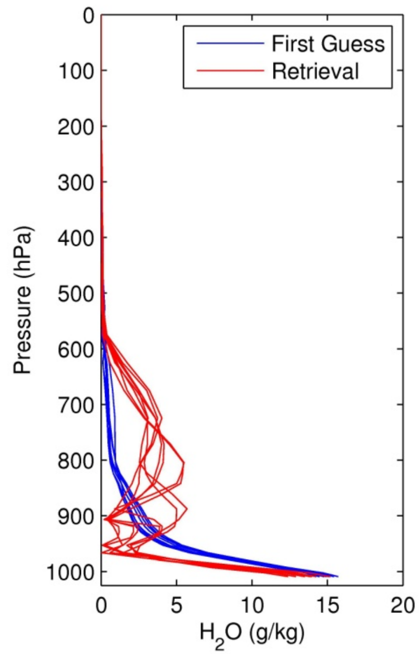


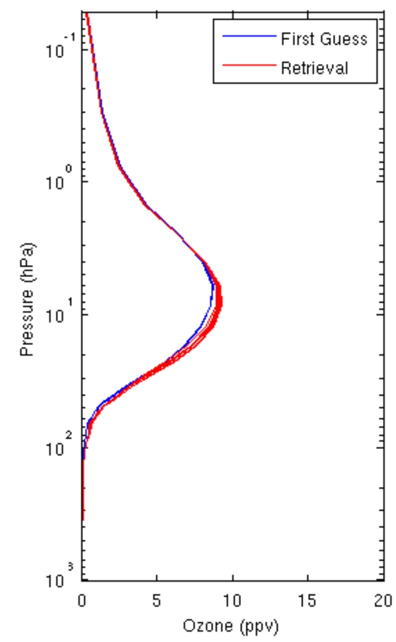
ε in LW lower than in the SW.
It means fine grain of sands

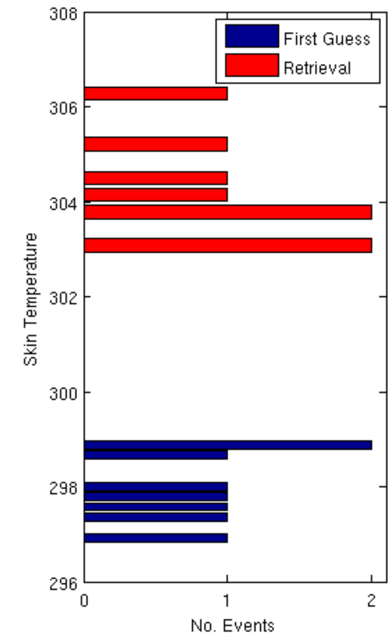
Simultaneously retrieved with ε(σ): T_s, T, H_2O, O_3



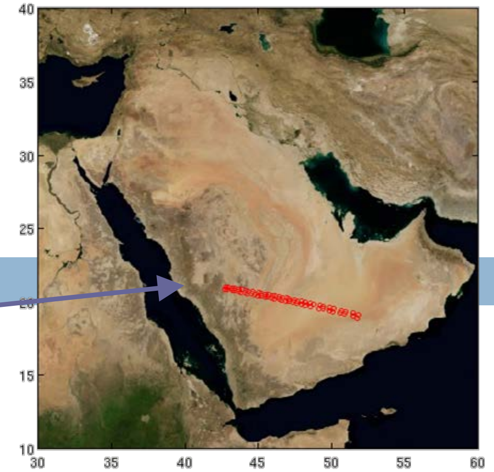




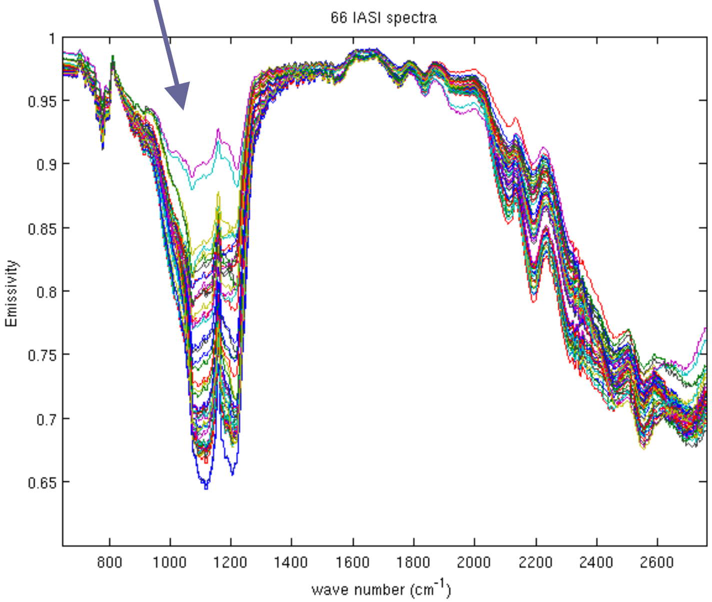


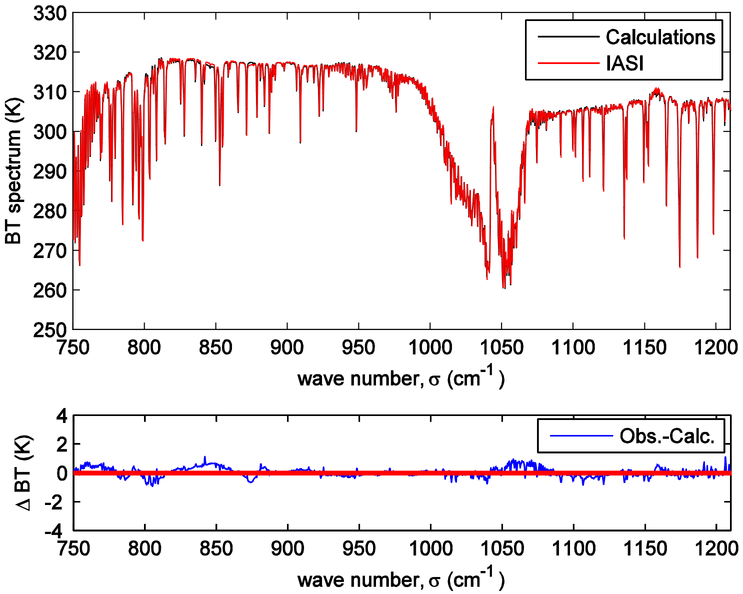


Arabian desert

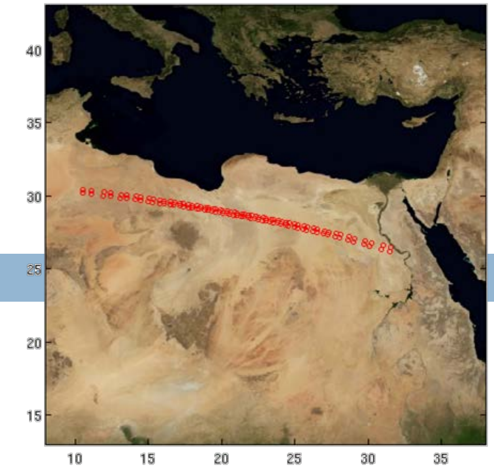


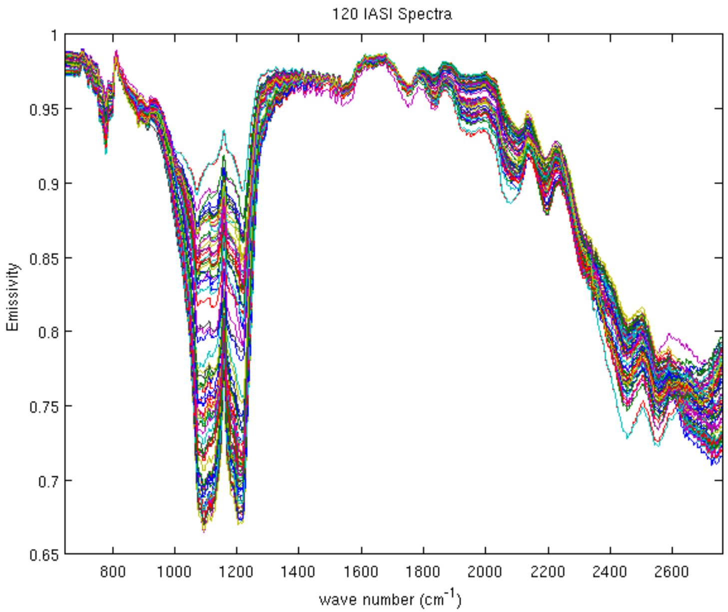
Coarse pixels (Western)

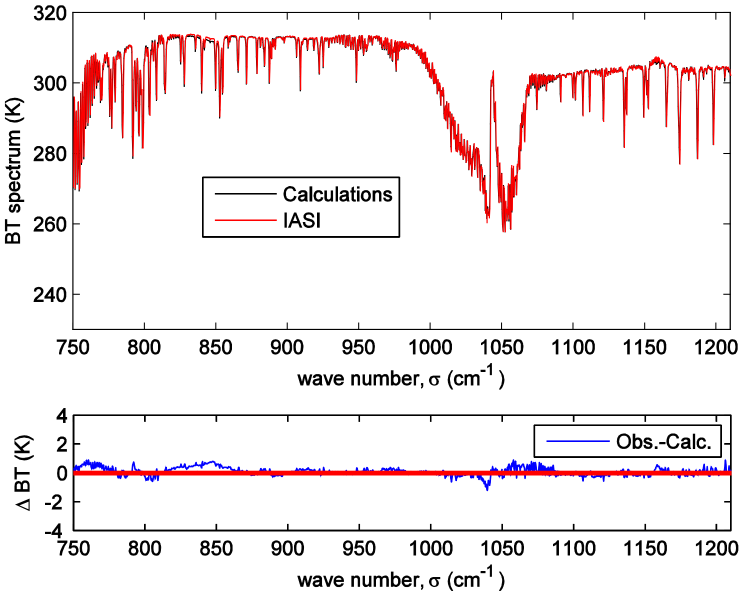




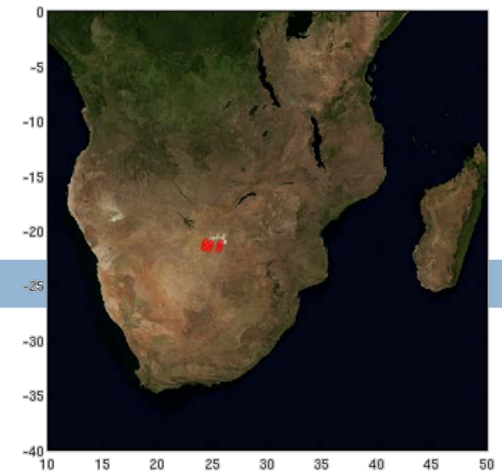
Sahara desert

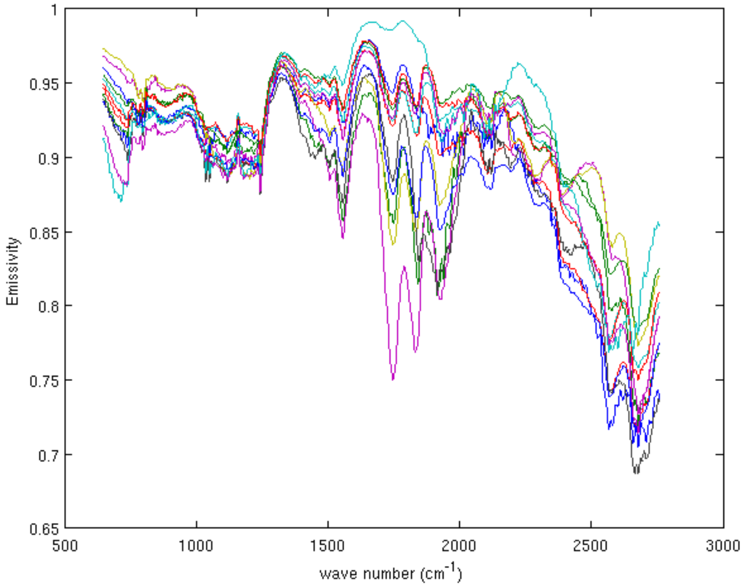


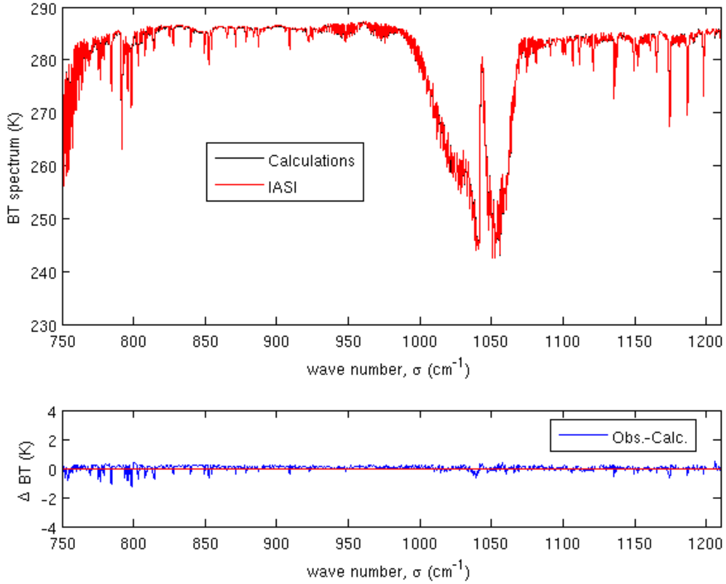




Kalahari Savanna







Conclusions

- We have developed a physical inverse methodology to retrieve the emissivity spectrum simultaneously with Surface Temperature and Atmospheric parameters: Temperature, water vapour and ozone. The methodology relies mainly on three basic ideas
  - Develop the emissivity spectrum in a truncated Fourier cosine series
  - Constrain the solution with Laboratory measurements
  - Balance the optimal estimation final product with a 2-Dimensional L-curve criterion
- A test retrieval exercise with IASI observations over desert area shows that the retrieved emissivity spectrum is capable to capture the fine details of the surface emission, even with a non committal background covariance matrix for emissivity
- The methodology will be soon applied to derive maps at global scale of the emissivity spectrum.

Set of parameters retrieved with *φ*-IASI

Simultaneously

- Emissivity spectrum
- Skin Temperature
- Temperature profile
- Water vapour profile
- Ozone profile

Sequentially, column amount

- CO
- CO_2
- CH_4
- N_2O