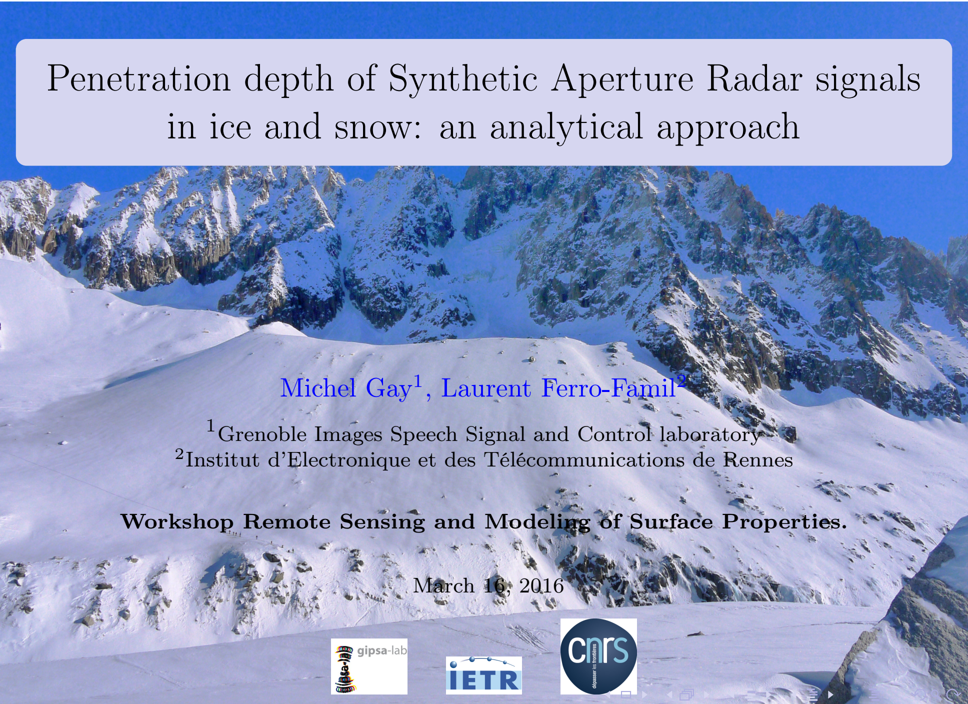


Penetration depth of Synthetic Aperture Radar signals in ice and snow: an analytical approach

Michel Gay[1], Laurent Ferro-Famil[2]

[1]Grenoble Images Speech Signal and Control laboratory
[2]Institut d'Electronique et des Télécommunications de Rennes

Workshop Remote Sensing and Modeling of Surface Properties.

March 16, 2016

Penetration depth δ_p

- δ_p, defines the depth within a medium at which the power of a propagating wave is equal to e^{-1} of its power at the medium's surface *(Ulaby et al. 1984)*.

- δ_p is a function of scattering and absorption losses within a medium, and can be calculated (*Ulaby et al. 1984, Drinkwater 1989*) by:

$$\delta_p = \frac{\lambda}{4\pi} \frac{1}{\sqrt{\{[1 + (\frac{\varepsilon''}{\varepsilon'})^2]^{\frac{1}{2}} - 1\} * \frac{\varepsilon'}{2}}} \tag{1}$$

λ : wavelength in free space

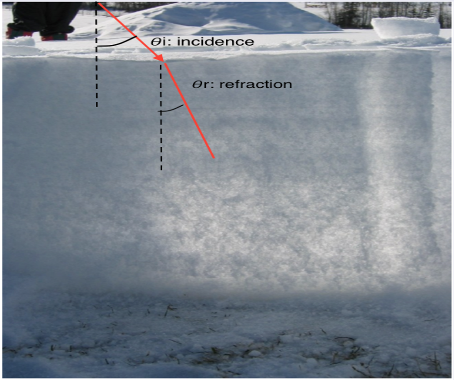
ε' and ε'': real part and imaginary part of the dielectric permittivity

- **δ_p represents the maximum depth within a medium that can contribute to the backscattering coefficient.**

Snowpack: complex stratified dense medium

- δ_p assumes an incident field perpendicular to the snow surface
- From satellite, the incidence angle θ_i is different from Nadir.
- Using Snell's law and $\varepsilon'' << \varepsilon'$: $\theta_i \approx \theta_r$

$$\delta_p' \approx \delta_p \; cos\theta_r \qquad (2)$$



Contents

Permittivity of ice ε_i

- Ice, unlike water, is a medium substantially transparent to microwave with a permittivity ε_i :

$$\varepsilon_i = \varepsilon_i' + i * \varepsilon_i'' \tag{3}$$

- In the field of microwaves, ε_i' is frequency independent and only slightly dependent of the temperature T.

- This dependence may be modeled by the formula given by *Mätzler et al.1987*

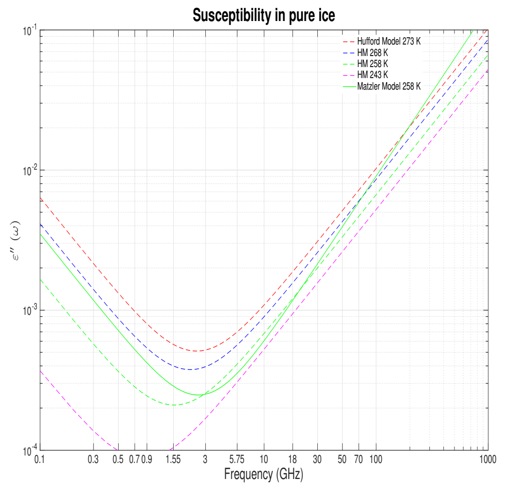
$$\varepsilon_i' = 3.1884 + 9.1 \ . \ 10^{-4}(T - 273\ K); \quad 243 \leq T \leq 273\ K \tag{4}$$

- The proposed model by *Hufford 1991* for ε_i'' is a good compromise between the theory of *Liebes's 1989* and the sets of data:

$$\varepsilon_i{}'' = \frac{\alpha}{f} + \beta\ f \tag{5}$$

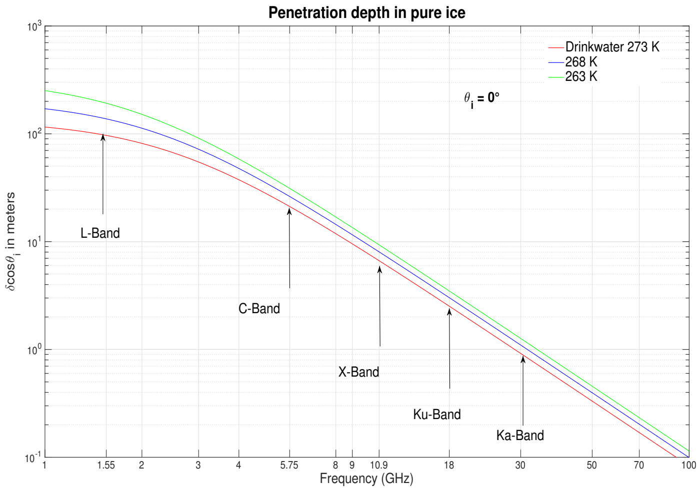
$\alpha = (0.00504 + 0.0062\ \theta)\ .\ exp(-22.1\ \theta)$
$\beta = \frac{0.502+0.131\ \theta}{1+\theta}\ 10^{-4} + 0.542\ 10^{-6}\ (\frac{1+\theta}{\theta+0.0073})^2$
with: $\theta = T_0/T - 1$ and $T_0 = 300\ K$ and f, the frequency

Permittivity of ice: ε_i''



- The imaginary part of permittivity ε'' decreases with the temperature.
- The permittivities have minimums between $0.9\ GHz$ and $2.9\ GHz$
- The Matzler model is slightly different.

Penetration depth: δ_p'



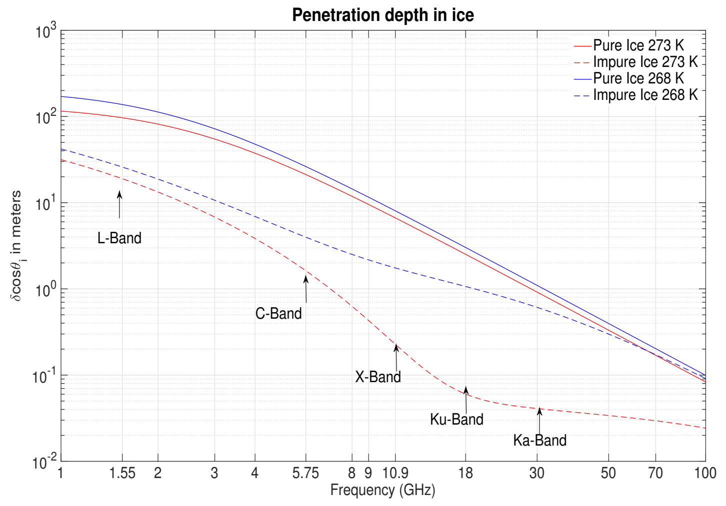
Penetration depths: δ'_p

Microwave penetration depth δ'_p $(\theta_i = 0°)$ in solid ice at three temperatures.

	1.55 GHz (m)	5.7 GHz (m)	10.9 GHz (m)	18.1 GHz (m)	30 GHz (m)
263 K	197	30	9.3	3.4	1.2
268 K	140	25	8.0	3.0	1.0
273 K	94	21	6.6	2.5	0.9

- The penetration depth decreases with increasing frequency.
- The temperature dependence of ice permittivity causes the penetration depth to decrease with temperature.
- At 30 GHz, this penetration depth is less than 2 m for $263 - 273$ K.

Penetration depth: δ'_p, with salinity $S = 35\ ppm$



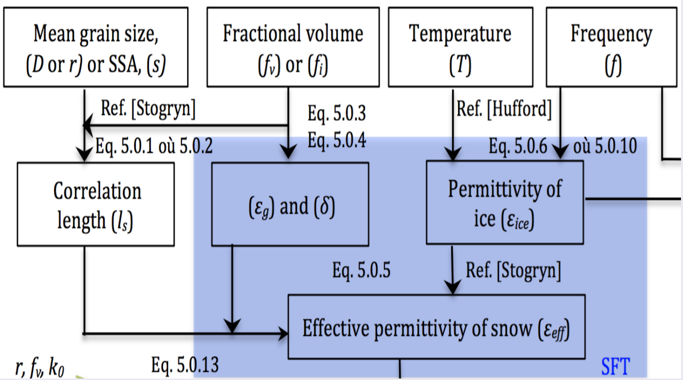
Permittivity of snow: ε_{eff}

We can perform the same type of calculation for snow

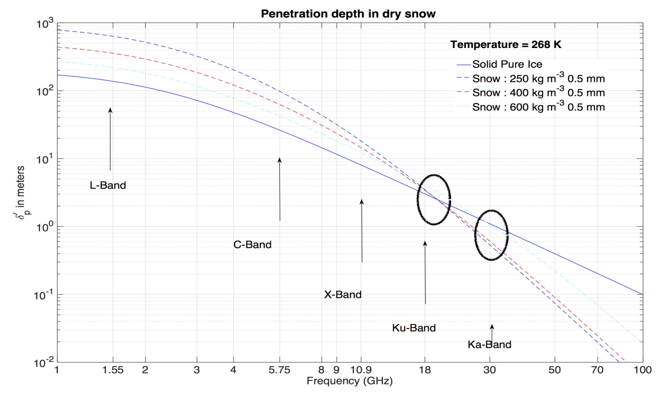
The complexe effective permittivity of snow ε_{eff} is given by:

$$\varepsilon_{eff} = \varepsilon_g + j.\frac{2}{3}\delta_{\varepsilon_g}.k_0^2 k_g \varepsilon_b.l_s^3 \tag{6}$$

with ε_g the quasi-static dielectric constant and δ_{ε_g} the variance of fluctuation

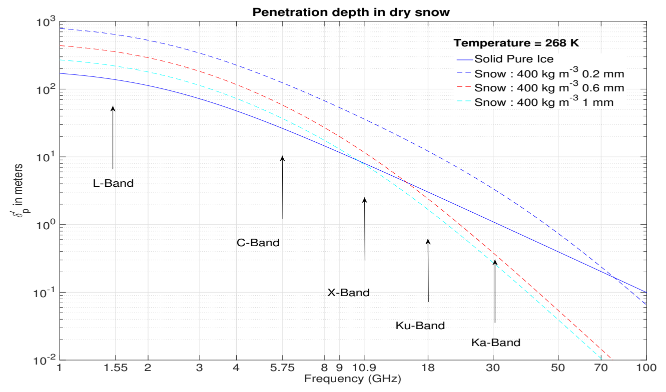


Penetration depth δ'_p in dry snow, density dependence



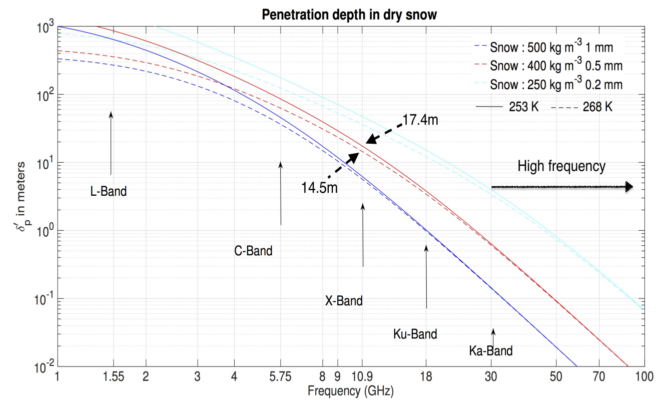
- The density determines the absorption losses (*Polder, Van Santen 1946*).
- Below around 18 GHz, the penetration depth is smallest for solid ice.
- Above around 30 GHz solid ice has the largest penetration depth -> scattering losses dominate absorption losses.
- Between 18 GHz and 30 GHz for this temperature and crystal size, absorption losses and scattering losses are nearly equal.

Penetration depth δ'_p in dry snow, crystal size dependence

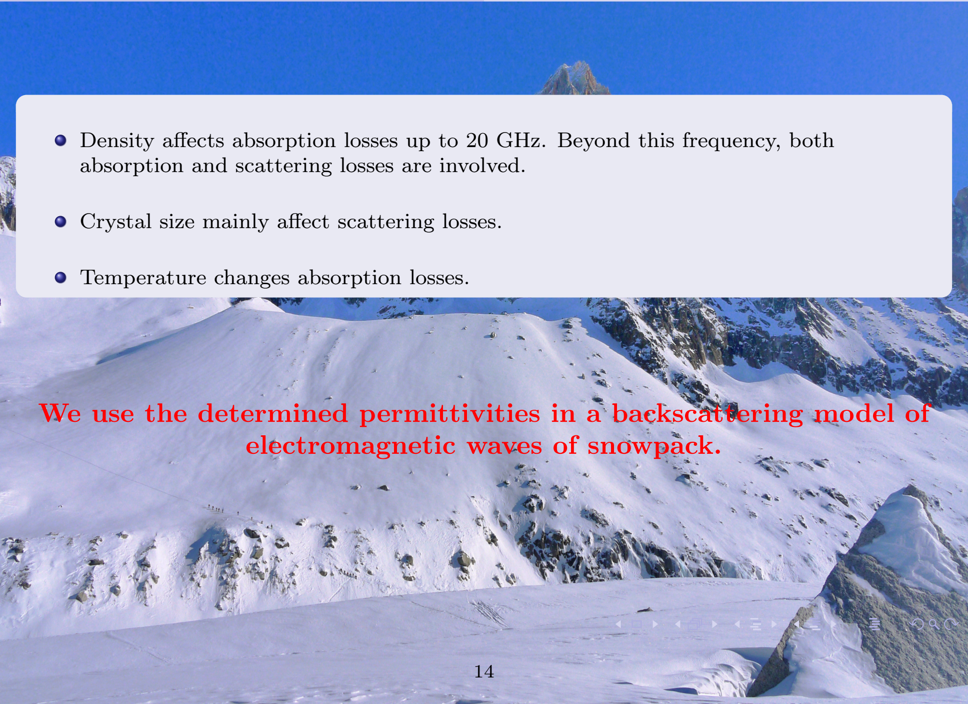


- Snow crystal size affects scattering losses.
- Scattering increases with the ratio of the crystal size to the microwave length.
- Example: scattering losses for the same snow type are larger at Ku-Band than X-band -> the penetration depth decreases.
- Larger crystal sizes have smaller penetration depths.

Penetration depth δ'_p in dry snow, temperature dependence



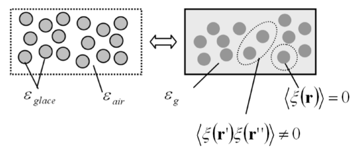
- δ'_p depends on snow type.
- For each snow type, in very high frequency, the dependence on temperature is not significant.
- Snow temperature changes are more significant below Ka-Band.
- Ex.: at X-band, snow type 2 has $\delta'_p \approx 17.4\ m$ for $253\ K$ and $\delta'_p \approx 14.5\ m$ for $268\ K$. The change being due to absorption losses.

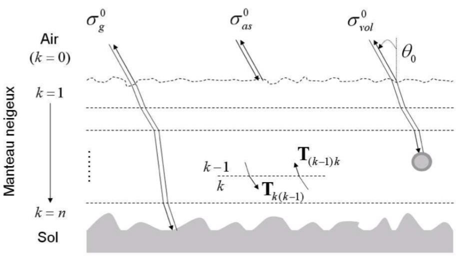


- Density affects absorption losses up to 20 GHz. Beyond this frequency, both absorption and scattering losses are involved.
- Crystal size mainly affect scattering losses.
- Temperature changes absorption losses.

We use the determined permittivities in a backscattering model of electromagnetic waves of snowpack.

Multilayer electromagnetic backscattering model *(L. Ferro-Famil, S. Allain, N. Longépé...)*





Three physical phenomena are taken into account to calculate σ_v^0 :

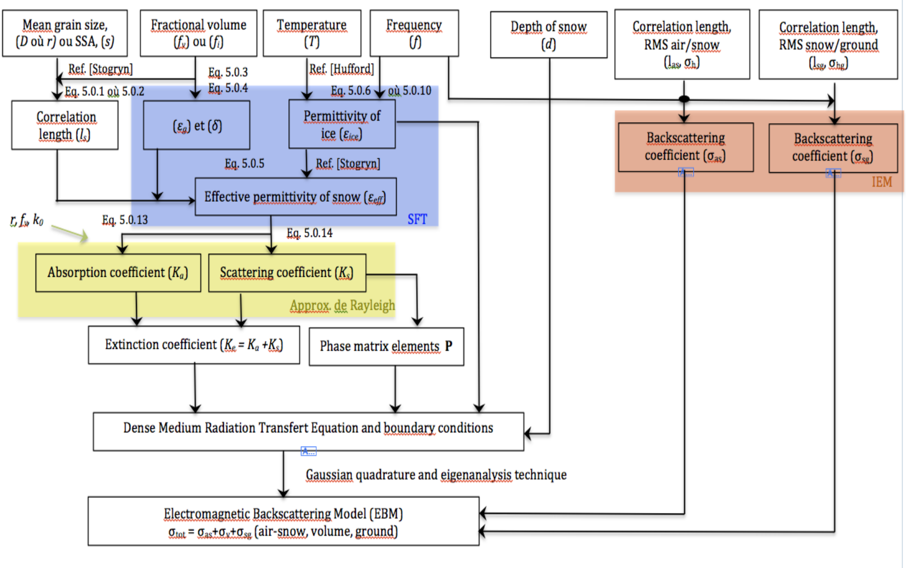
1. Attenuation : SFT -> ε_{eff} -> K_e
2. Scattering : SFT-> phase matrix
3. Refraction : transmission matrix $\mathbf{T}$

Calculation of σ_{sim}^0: sum of three coefficients

$$\boldsymbol{\sigma}^{\mathbf{0}}_{\mathbf{sim}} = \boldsymbol{\sigma}^{\mathbf{0}}_{\mathbf{as}} + \boldsymbol{\sigma}^{\mathbf{0}}_{\mathbf{v}} + \boldsymbol{\sigma}^{\mathbf{0}}_{\mathbf{g}} \qquad (7)$$

σ_v^0: DMRT *(Longépé et al., 2009)*

σ_{as}^0 and σ_g^0: IEM *(Fung et al., 2010)*.



Mean grain size, (D où r) ou SSA, (s)

Fractional volume (f_v) ou (f_i)

Temperature (T)

Frequency (f)

Depth of snow (d)

Correlation length, RMS air/snow (l_{as}, σ_h)

Correlation length, RMS snow/ground (l_{sg}, σ_{hg})

Ref. [Stogryn]

Eq. 5.0.1 où 5.0.2

Eq. 5.0.3

Eq. 5.0.4

Ref. [Hufford]

Eq. 5.0.6 où 5.0.10

Correlation length (l_c)

(ε_g) et (δ)

Permittivity of ice (ε_{ice})

Backscattering coefficient (σ_{as})

Backscattering coefficient (σ_{sg})

IEM

Eq. 5.0.5

Ref. [Stogryn]

Effective permittivity of snow (ε_{eff})

SFT

r, f_v, k_0

Eq. 5.0.13

Eq. 5.0.14

Absorption coefficient (K_a)

Scattering coefficient (K_s)

Approx. de Rayleigh

Extinction coefficient ($K_e = K_a + K_s$)

Phase matrix elements **P**

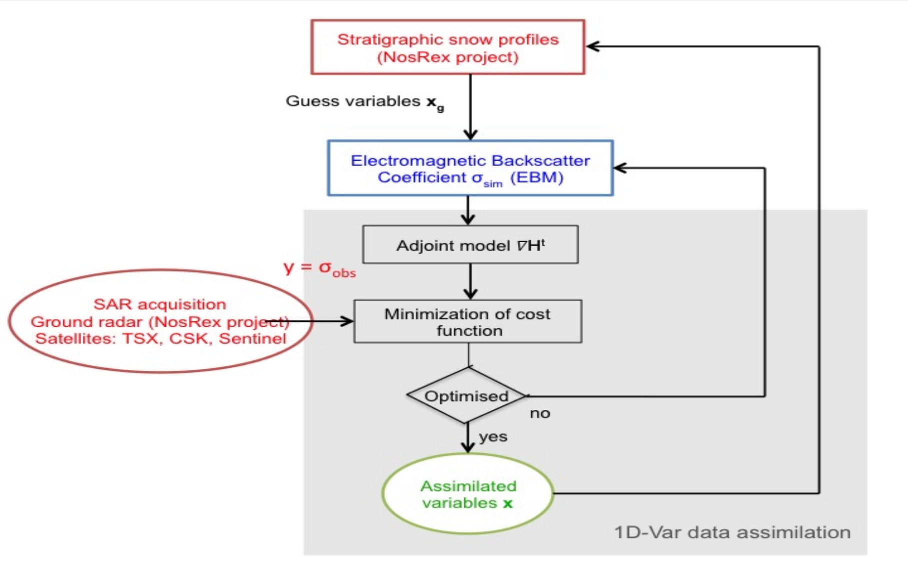
Dense Medium Radiation Transfert Equation and boundary conditions

Gaussian quadrature and eigenanalysis technique

Electromagnetic Backscattering Model (EBM)

$\sigma_{tot} = \sigma_{as} + \sigma_v + \sigma_{sg}$ (air-snow, volume, ground)

Assimilation algorithm SAR data *(X.V. Phan)*.

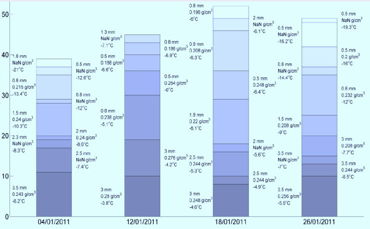


Sodankyla: ground Radar, stratigraphic profiles, TerraSAR-X acquisition

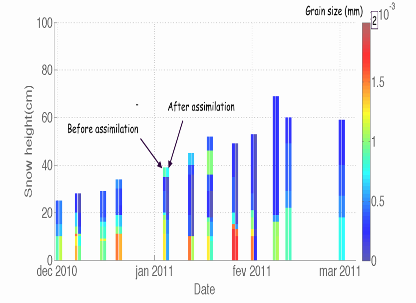
Ground Radar observation-> SnowScat instrument

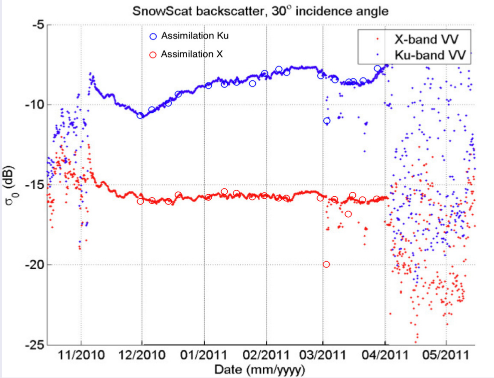
	Parameters
Frequency	9.2-17.8 GHz
Incidence	$30^o < \theta < 60^o$
Polarisation	HH, HV, VH, VV

Stratigraphic snow profiles



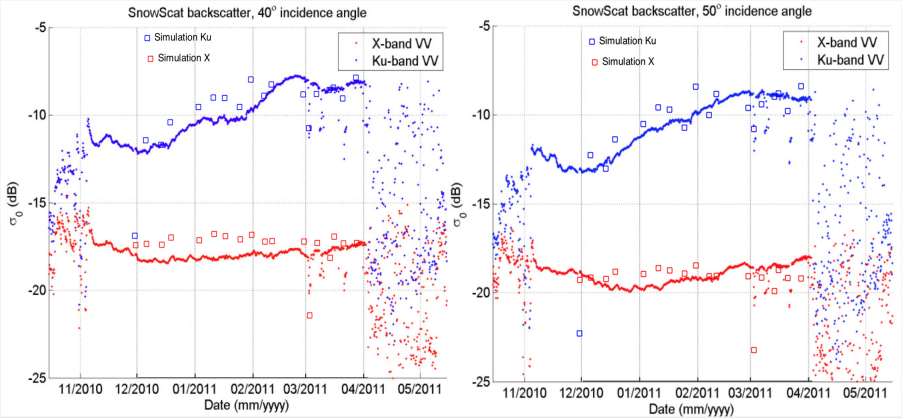
With our assimilation algorithm we can modify the observed stratigraphic profiles of snow (incidence angle $\theta_i = 30°$).





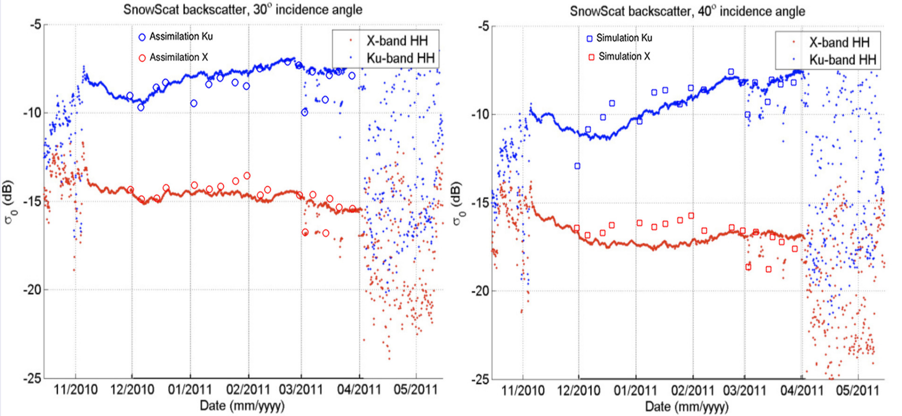
- Backscatter coefficients calculated $\sigma_{sim}=\sigma_{as}+\sigma_{vol}+\sigma_{sg}$ converge well to the values measured by the ground radar at $Ku-band$ $(16.7\ GHz)$ and $X-band$ $(10.2\ GHz)$

New stratigraphic snow profiles are used to simulate the backscattering σ_{sim} at incidence angle $\theta_i = 40^\circ$ and $\theta_i = 50^\circ$.

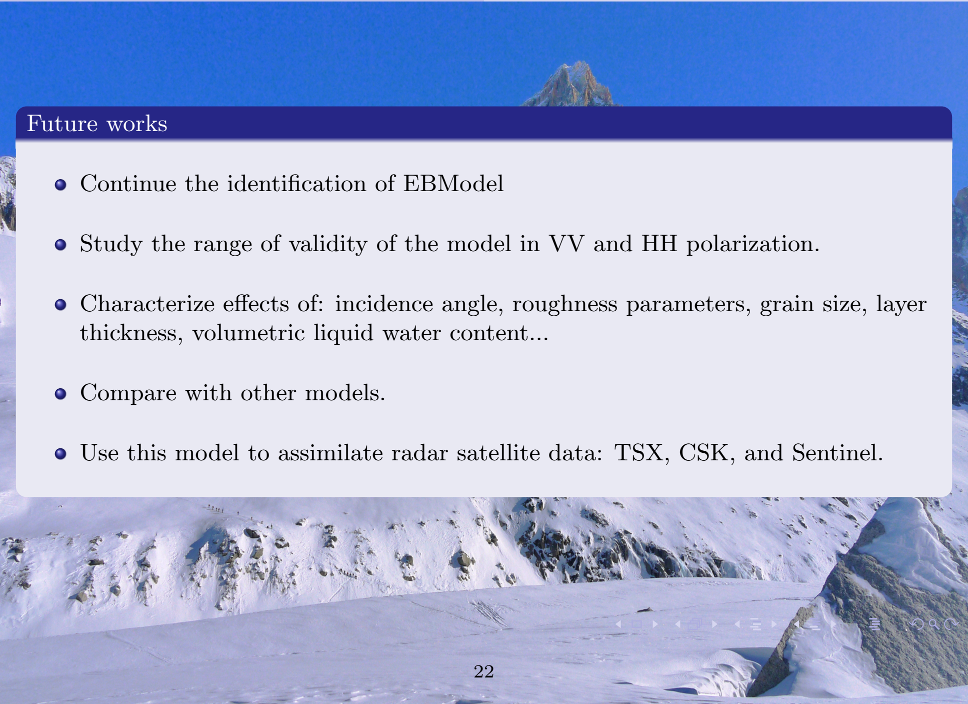


- We have a good agreement between the backscatter coefficients calculated and measured at X-Band and Ku-Band.
- At these two frequencies in VV polarization, the most significant contributions to the variation in backscattering are grain size and roughness of snow ground interface.

In the same way, we can assimilate the backscattering σ_{assim} in polarization HH at incidence angle $\theta_i = 30°$ and simulate the backscattering σ_0 at $\theta_i = 40°$.



- Again, we have a relative good agreement between the backscatter coefficients calculated and measured at X-Band and Ku-Band.



Future works

- Continue the identification of EBModel
- Study the range of validity of the model in VV and HH polarization.
- Characterize effects of: incidence angle, roughness parameters, grain size, layer thickness, volumetric liquid water content...
- Compare with other models.
- Use this model to assimilate radar satellite data: TSX, CSK, and Sentinel.

Thank you for your attention

Publications

- *M. Gay, X.V. Phan, L. Ferro-Famil, F. Karbou, Y. Durand, G. D'Urso, and A. Girard. Simulation de la rétrodiffusion radar du manteau neigeux, comparaison avec les données d'un radar sol et TSX (projet NoSRex) ENVIREM, Paris 2015.*
- *X.V. Phan, L. Ferro-Famil, M. Gay, Y. Durand, S. Morin, S. Allain, G. D'Urso, and A. Girard, 1D-Var multilayer assimilation of X-band SAR data into a detailed snowpack model. Cryopsphere, 8, p.1975-1987, 2014.*
- *X.V. Phan, L. Ferro-Famil, M. Gay, Y. Durand, M. Dumont. Comparaison between DMRT simulations for multilayer snowpack and data from NOSREX project. IGARSS 2014.*
- *L. Ferro-Famil, C. Leconte, F. Boutet, X. V. Phan, M. Gay, and Y. Durand. PoSAR: a VHR tomographic GB-SAR system Application to snow cover 3-D imaging at X and Ku Bands, in 9th European Radar Conference (EuRAD 2012), (Amsterdam, Pays-Bas), Oct. 2012*