## Optical Path Transmittance: OPTRAN. Forward and Adjoint Modeling

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## Traditional Fast Transmittance Model

- Interpolate T(P), q(P) to fixed pressure levels
- Predictors T, q
- Include zenith angle as a predictor
- Predictand is transmittance departure or optical depth, multiple linear regression

## Optical Path Transmittance (OPTRAN) approach

- Regression on levels of absorber amount
- Predictors are a function of T, P, q
- Zenith angle implicit in absorber amount
- Arbitrary pressure profile permitted
- Predictand is absorption coefficient for H2O, O3, mixed gases
- Permits changes to 'mixed gas' amounts as well

#### Heritage

• McMillin, Fleming and Hill (AO,1979)

• McMillin, Crone, Goldberg, Kleespies (AO,1995)

• McMillin, Crone, Kleespies (AO,1995)

• Three papers in the works

## **OPTRAN** performance

- Water vapor channel much better than RTTOV
- Temperature channels generally not quite as good as RTTOV (before McMillin improvements)

#### What's this adjoint stuff it all about?

1DVAR / maximum probability solution is that which minimizes a 'cost' or 'penalty function:

$$\mathbf{J}(\mathbf{x}) = (\mathbf{x} - \mathbf{x}^{\mathrm{b}})^{\mathrm{T}} \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}^{\mathrm{b}}) + (\mathbf{y}^{\mathrm{o}} - \mathbf{y}(\mathbf{x}))^{\mathrm{T}} \mathbf{O}^{-1} (\mathbf{y}^{\mathrm{o}} - \mathbf{y}(\mathbf{x}))$$

where  $\mathbf{x}^{b}$  is an initial estimate given by the model state vector,  $\mathbf{x}$  is the model state for which the solution is desired,  $\mathbf{y}^{o}$  is the vector of observations,  $\mathbf{y}(\mathbf{x})$  is an operator which transforms the model state vector into the same parameters as the observations, and  $\mathbf{B}$  and  $\mathbf{O}$  are the background and observational error covariance matrices respectively. For our purposes,  $\mathbf{y}(\mathbf{x})$  is the radiative transfer operator. Note that  $\mathbf{O}$ is a combination of observational errors and radiative transfer errors. (This is just a least squares problem)

#### What's it all about: part deux

How do we find the minimum? From first quarter Calculus: Take the first derivative and set it equal to zero.

$$\nabla \mathbf{J}(\mathbf{x}) = \mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}^{\mathrm{b}}) - \mathbf{K}(\mathbf{x})^{\mathrm{T}} \mathbf{O}^{-1}(\mathbf{y}^{\mathrm{o}} - \mathbf{y}(\mathbf{x})) = 0$$

where  $\mathbf{K}(\mathbf{x})$  is the matrix of partial derivatives of  $\mathbf{y}(\mathbf{x})$  with respect to the elements of  $\mathbf{x}$ . (factor of 2 divides out)

#### What's it all about: part trois

It is evident that the solution requires both the forward radiative transfer operator y(x), and the transpose of its derivative,  $K(x)^T \cdot K(x)^T$  is called the adjoint, or Jacobian.

$$\mathbf{x} = \{ \mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3, \dots, \mathbf{T}_n, \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \dots, \mathbf{q}_n, \dots \}$$
$$\mathbf{y}(\mathbf{x}) = \{ \mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3, \dots, \mathbf{R}_m \}^{\mathrm{T}}$$

#### What's it all about, part quatre

$$\mathbf{K}(\mathbf{x})^{\mathrm{T}} = \begin{bmatrix} \frac{\partial \mathbf{R}_{1}}{\partial \mathbf{T}_{1}} & \frac{\partial \mathbf{R}_{2}}{\partial \mathbf{T}_{1}} & \frac{\partial \mathbf{R}_{3}}{\partial \mathbf{T}_{1}} & \cdots & \frac{\partial \mathbf{R}_{\mathrm{m}}}{\partial \mathbf{T}_{1}} \\ \frac{\partial \mathbf{R}_{1}}{\partial \mathbf{T}_{2}} & \frac{\partial \mathbf{R}_{2}}{\partial \mathbf{T}_{2}} & \frac{\partial \mathbf{R}_{3}}{\partial \mathbf{T}_{2}} & \cdots & \frac{\partial \mathbf{R}_{\mathrm{m}}}{\partial \mathbf{T}_{2}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \mathbf{R}_{1}}{\partial \mathbf{T}_{\mathrm{n}}} & \frac{\partial \mathbf{R}_{2}}{\partial \mathbf{T}_{\mathrm{n}}} & \frac{\partial \mathbf{R}_{3}}{\partial \mathbf{T}_{\mathrm{n}}} & \cdots & \frac{\partial \mathbf{R}_{\mathrm{m}}}{\partial \mathbf{T}_{\mathrm{n}}} \\ \frac{\partial \mathbf{R}_{1}}{\partial \mathbf{q}_{1}} & \frac{\partial \mathbf{R}_{2}}{\partial \mathbf{q}_{1}} & \frac{\partial \mathbf{R}_{3}}{\partial \mathbf{q}_{1}} & \cdots & \frac{\partial \mathbf{R}_{\mathrm{m}}}{\partial \mathbf{q}_{1}} \\ \frac{\partial \mathbf{R}_{1}}{\partial \mathbf{q}_{2}} & \frac{\partial \mathbf{R}_{2}}{\partial \mathbf{q}_{2}} & \frac{\partial \mathbf{R}_{3}}{\partial \mathbf{q}_{2}} & \cdots & \frac{\partial \mathbf{R}_{\mathrm{m}}}{\partial \mathbf{q}_{2}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \mathbf{R}_{1}}{\partial \mathbf{q}_{\mathrm{n}}} & \frac{\partial \mathbf{R}_{2}}{\partial \mathbf{q}_{\mathrm{n}}} & \frac{\partial \mathbf{R}_{3}}{\partial \mathbf{q}_{\mathrm{n}}} & \cdots & \frac{\partial \mathbf{R}_{\mathrm{m}}}{\partial \mathbf{q}_{\mathrm{n}}} \end{bmatrix}$$

#### What's it all about, part cinq

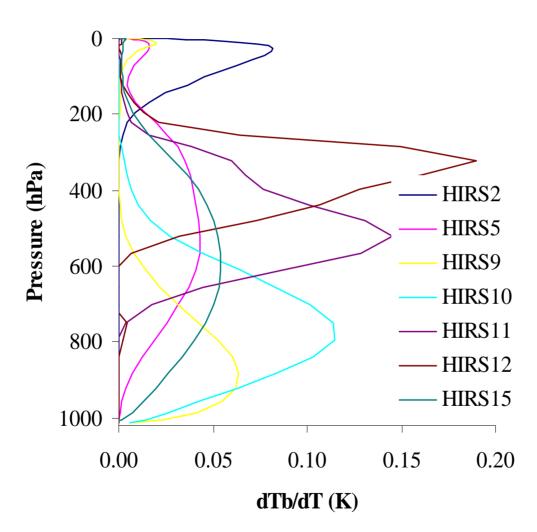
In olden days (say 1990), computation of  $\mathbf{K}(\mathbf{x})^{T}$  required N+1 forward model calculations using forward (or backward) finite differencing (centered required 2N+1). Thus these techniques were only used in limited studies

In these modern times, using adjoint coding techniques  $\mathbf{K}(\mathbf{x})^{T}$  can be computed with the effort of about 3 forward model calculations.

#### What are these all the models?

- The tangent linear model is derived from the forward model
  - gives the derivative of the radiance with respect to the state vector (vector output, m channels)
- The adjoint is derived from the tangent linear model
  - gives the transpose of the derivative of the radiance with respect to the state vector (vector output, N variables)
- The Jacobian is derived from the adjoint model
  - gives the transpose of the derivative of the radiance with respect to the state vector by channel (matrix output, Nxm)
- At NCEP, only the forward and the Jacobian models are actually used, but all models must be developed and maintained in order to assure a testing path, and to make sure the performance is correct.

**Tropical Temperature derivatives for NOAA 14** 



# Why can't we just use the Tangent Linear Model?

- You can.
- However, it still takes N TL calculations.
- You avoid the finite differencing because the TL is the analytic derivative, but you just get a vector of radiances for each call. You still have to call it for each element of the input vector.

## Testing

- Testing the code is rigorous and analytic
- Each code is tested for consistency with the model from which it was developed
- Code is tested bottom up, lowest level first.
- Full TL model is tested before moving to adjoint
- Full Adjoint is tested before moving to Jacobian

### Adjoint Compiler

- Giering and others have written compilers that generate TL and adjoint code
- Some people at NCAR swear by them
- Others swear at them (just kidding)
- We feel that better optimization can be achieved by hand coding.

### Summary

• Quick overview of OPTRAN

• Description of Adjoint and associated models

• Keep these brave souls who will take the coding class in your thoughts.

## Class Participants Please Remain for a Few Minutes