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Reconstruction of surface profiles by iterative Newton-Kantorovitch's method

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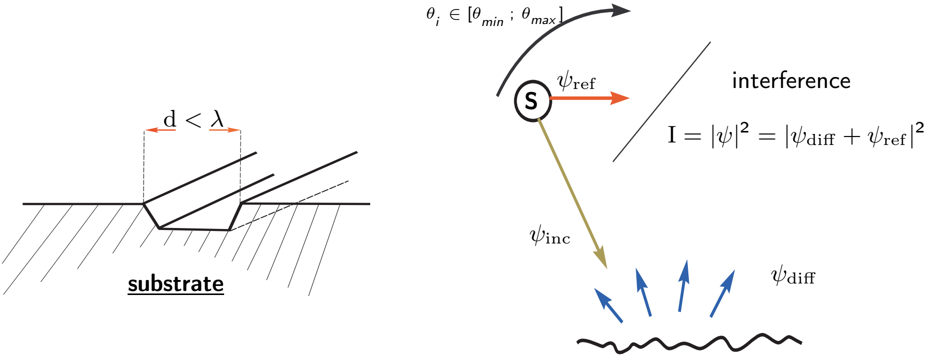
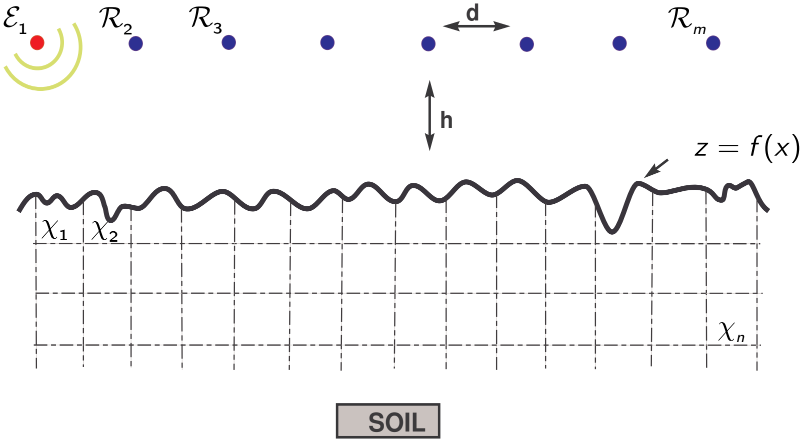
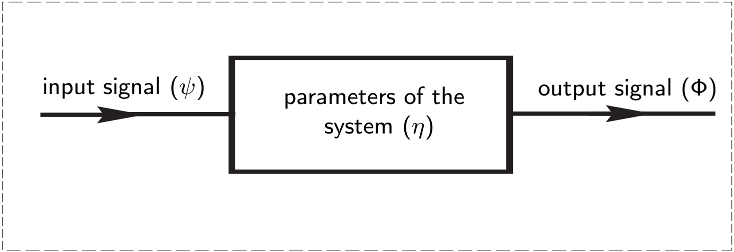


Objective : characterize surface roughness with lateral resolution beyond the Rayleigh criterion



$\eta = \{f, \chi\}$. for a known contrast, $\eta = f$



$$\begin{cases} \psi \in \mathcal{E}, & \text{space of the input signal} \\ \Phi \in \Sigma, & \text{space of the output signal} \\ \eta \in \Gamma, & \text{parameter space of the system} \end{cases}$$

$\mathbf{F}(\eta)$ an operator that depends on the parameter η, such as : $\Phi = \mathbf{F}(\eta)\,\psi$

in general ψ is well known, is noted for brevity : $\Phi = \mathbf{F}(\eta)$

Example (TE ; Dirichlet).

Observation equation

$$\Phi(\theta,\theta_i) \propto -\int_\Gamma \partial_n\psi(x,\eta(x),\theta_i)\ e^{-i(k_x(\theta)\,x+k_z(\theta)\,\eta(x))}\ d\Gamma$$

State equation

$$\psi_{inc}(x',\eta(x'),\theta_i) = \int_\Gamma g(x,\eta(x),x',\eta(x'))\ \partial_n\psi(x,\eta(x),\theta_i)\ d\Gamma$$

$g(x,\eta(x),x',\eta(x')) = \frac{i}{4}\, H_0^1(k\sqrt{(x-x')^2+(\eta(x)-\eta(x'))^2})$: free space Green's function.

1. L. Tsang, J. Kong, K. Ding, C. Ao "Scattering of electromagnetic waves" 2001 by John Wiley and Sons, Inc.

- Solving the forward problem is to :

$$\boxed{\text{For } \eta \in \Gamma, \text{ we calculate : } \Phi = \mathbf{F}(\eta)}$$

- Solving the inverse problem is to :

$$\boxed{\text{For } \Phi \in \Sigma, \text{ we calculate : } \eta = \mathbf{F}^{-1}(\Phi)}$$

- Note : This inverse problem is difficult to solve because it is ill-posed, meaning , it does not satisfy at least one of the three Hadamard's criteria, defined for a well-posed problem :
  - existence of the solution
  - uniqueness of the solution
  - continuity of the solution against data

+ nonlinearity of **F** ! ! !

Fréchet's Operator

Definition : Newton-Kantorovich's method is to find the Fréchet's operator **D**, that linearise locally (in the vicinity of η) the nonlinear operator **F** :

$$\mathbf{F}(\eta + \delta\eta) = \mathbf{F}(\eta) + \mathbf{D}\delta\eta + o(\|\delta\eta\|_2), \text{ lorsque } \|\delta\eta\|_2 \to \mathbf{0}$$

$$\mathbf{F}(\eta + \delta\eta) - \mathbf{F}(\eta) = \delta\Phi \simeq \mathbf{D}\delta\eta$$

$$\delta\Phi = \mathbf{D}\,\delta\eta$$

The linearised problem remains ill-posed

$$\xi = ||\delta\Phi - \mathbf{D}\,\delta\eta||_2^2$$

$$+$$

$$\Pi(\delta\eta) = \sum_{r=0}^{p} \int_\Gamma q_r(x) \left(\frac{d^r\delta\eta}{dx^r}\right)^2 dx$$

Tikhonov regularisation

$$\tilde{\xi} = ||\delta\Phi - \mathbf{D}\,\delta\eta||_2^2 + \mu^2\,\Pi(\delta\eta)$$

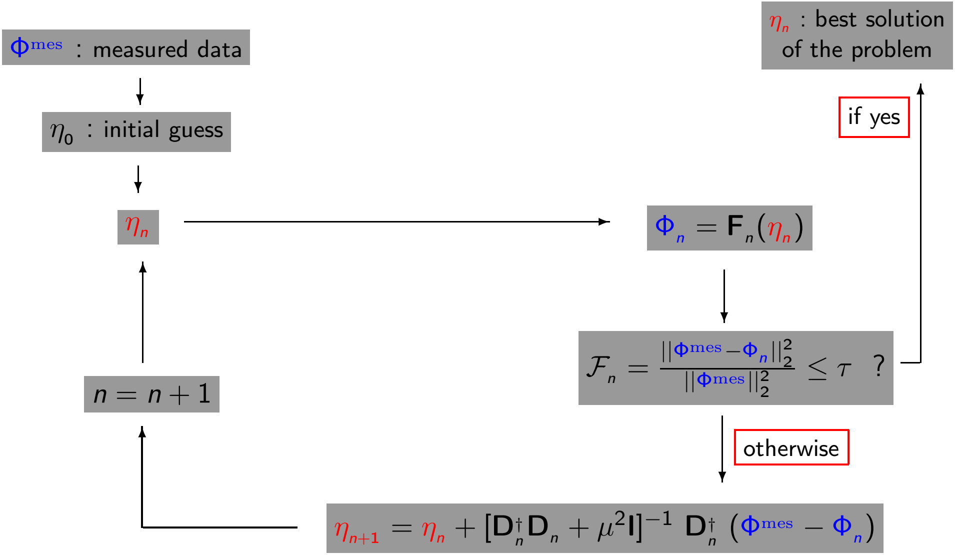
Least squares (ex : $\Pi(\delta\eta) = ||\delta\eta||_2^2$)

$$\delta\eta = [\mathbf{D}^\dagger\mathbf{D} + \mu^2\mathbf{I}]^{-1}\mathbf{D}^\dagger\;\delta\Phi$$

I : identity operator

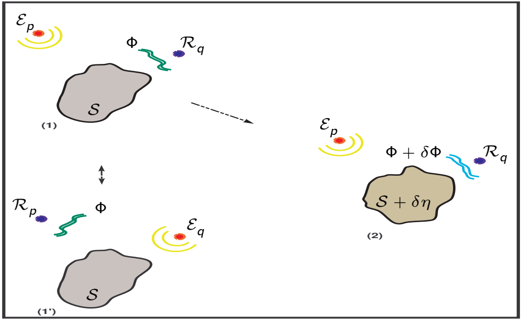
μ^2 : Tikhonov parameter

Organigram of the inversion algorithm



For a small change $\delta\eta$ in the surface $\mathcal{S}$, what change $\delta\Phi$ in the scattered field ?

The Fréchet's operator **D** stands as a linear relationship between $\delta\eta$ and $\delta\Phi$, such as : $\delta\Phi = \mathbf{D}\,\delta\eta$



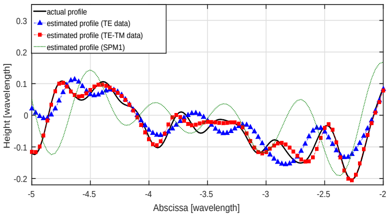
surface profile (perfectly conducting object),

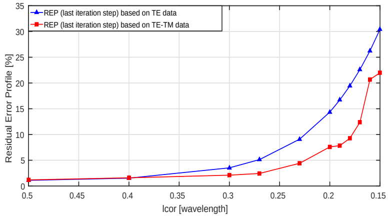
$$\delta\Phi \propto \int_{\mathcal{S}} [\,\mathrm{k}^{2}\,\mathbf{J}_{(1)}.\mathbf{J}_{(1')} - (\nabla_s.\mathbf{J}_{(1)})(\nabla_s.\mathbf{J}_{(1')})\,]\,\delta\eta\;\mathrm{d}\mathcal{S}$$

surface profile (dielectric object),

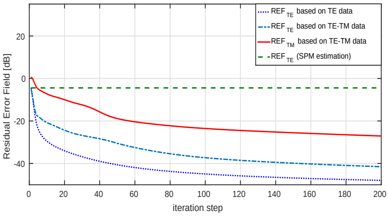
$$\delta\Phi \propto \int_{\mathcal{S}} \chi\,[\,(\mathbf{E}^n_{(1)} + \mathbf{E}^t_{(1)}).(\tfrac{1}{1+\chi}\mathbf{E}^n_{(1')} + \mathbf{E}^t_{(1')})\,]\,\delta\eta\;\mathrm{d}\mathcal{S}$$

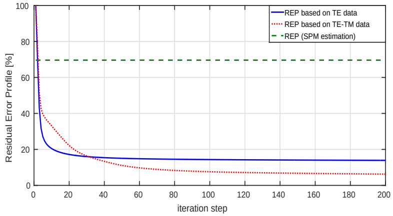
2. A. Roger Electromagnetics 2 :69-83, 1982.





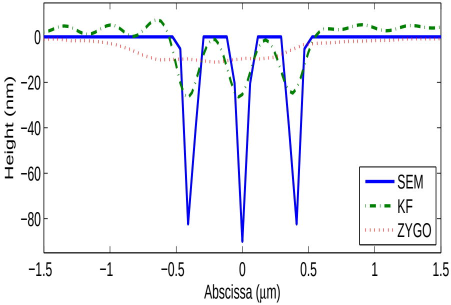
$hrms = 0.095\ \lambda, \ lcor = 0.2\ \lambda$

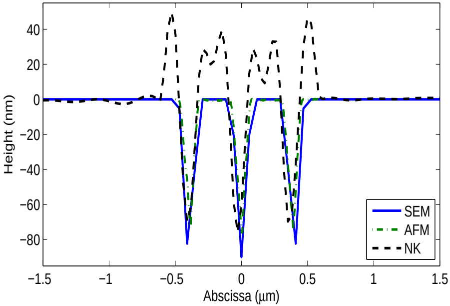




S. Arhab et al. "Inverse wave scattering of rough surfaces with emitters and receivers in the transition zone"
Progress In Electromagnetics Research M, Vol. 45, 131–141, (2016).

$R_c = 223$ nm





Indium phosphide surface (InP) : $\varepsilon = 12.37 + 2.11\,i$

S. Arhab et al. "Nanometric resolution with far-field optical profilometry" Phys. Rev. Lett. 111, 053902 (2013)

THANK YOU FOR YOUR ATTENTION.